(2.7) In one mole of gas at STP (standard temperature and pressure, i.e. 0 deg.C and 1 atm) a small fraction of the molecules have a kinetic energy >= 15***k**T*. (a) How many such molecules are there, and (b) what would their temperature be if they could be isolated?

As usual we begin by defining the values of the electron volt, *eV*, the Boltzmann constant, *k*, and the Avogadro number, N_A using data from Appendix III and IV:

$eV := 1.6021773 \cdot 10^{-19} \cdot joule$ $\quad k := 1.38066 \cdot 10^{-23} \cdot eV \cdot K^{-1}$ $\quad N_A := 6.022137 \cdot 10^{23} \cdot mole^{-1}$

Then we convert the temperature, *T*, from deg.C to deg.K as follows:

$T := (0 + 273.15) \cdot K$

(a) The number of ***k**T* units, *x*, is given and thus:

$x := 15$

In order to avoid numerical problems we only integrate eqn. (2.25) up to 250***k**T* as a substitute for infinity (otherwise the computer will experience an underflow exception):

$$f := \int_{x \cdot k \cdot T}^{250 \cdot k \cdot T} \frac{2 \cdot \sqrt{E}}{\sqrt{\pi} \cdot (k \cdot T)^{\frac{3}{2}}} \cdot exp\left(-\frac{E}{k \cdot T}\right) dE$$

eqn (2.25) without any rearrangements

$f = 3.002 \cdot 10^{-6}$ $\quad f = 3.002 \cdot 10^{-4} \cdot \%$

Answer is : $\quad f \cdot N_A = 1.808 \cdot 10^{18}$ $\quad$ molecules per mole

(b) In order to calculate the corresponding temperature we begin by defining the integrand of eqn. (2.25) with the unknown energy (designated by *z*) as a function of *z*:

$$g(z) := \frac{2 \cdot \sqrt{z}}{\sqrt{\pi} \cdot (k \cdot T)^{\frac{3}{2}}} \cdot exp\left(-\frac{z}{k \cdot T}\right)$$

We then use eqn (2.21) to calculate the temperature from the average energy, E_{tr}:

$$E_{tr} := \frac{\int_{x \cdot k \cdot T}^{250 \cdot k \cdot T} E \cdot g(E)\, dE}{\int_{x \cdot k \cdot T}^{250 \cdot k \cdot T} g(E)\, dE}$$

(This is the standard method to calculate the average of a function, in this case of *E*, over an interval)

$T_{calc} := \frac{2}{3 \cdot k} \cdot E_{tr}$ $\quad T_{calc} = 2737 \cdot K$