(2.11) A gas centrifuge plant is set up in order to enrich UF_6 of natural isotopic composition in ^{235}U. The centrifuges, which each have a length of 100 cm and a diameter of 20 cm, rotate at 40000 rpm. The gas temperature is 70 deg.C.

(a) Prove that the separation factor α in (2.47) can be approximated by e^{δ} according to (2.58) when the product flow is very small compared to the waste flow, and α is not far from 1.

$e^{\delta} = 1 + \delta + \delta^2/2 + \ldots$ But $\alpha = 1 + \delta$. Neglecting higher order terms we get $e^{\delta} = \alpha$. Q.E.D.

(b) Using this approximation, what is the theoretical separation factor for one unit?

$u := 1.660540 \cdot 10^{-27} \cdot kg$ $R := 0.08206 \cdot liter \cdot atm \cdot mole^{-1} \cdot K^{-1}$ $T := (70 + 273.15) \cdot K$

$M_H := (238 + 6 \cdot 19) \cdot \frac{gm}{mole}$ $M_L := (235 + 6 \cdot 19) \cdot \frac{gm}{mole}$

$\omega := 2 \cdot \pi \cdot \frac{40000}{60} \cdot sec^{-1}$ $r := \frac{20}{2} \cdot 0.01 \cdot m$ $v_{rot} := \omega \cdot r$ $v_{rot} = 418.879 \cdot m \cdot sec^{-1}$

$\delta := \left(M_H - M_L\right) \cdot \omega^2 \cdot \frac{r^2}{R \cdot T \cdot 2}$ OBS! 1/1000 omitted because of automatic unit conversion used.

$\delta = 0.092$ $\alpha := exp(\delta)$ $\alpha = 1.097$

(c) Assuming that the enrichment factor obtained with the centrifuge is only 70% of the theoretical one, what number of units would be required in series in order to achieve UF_6 with 3% ^{235}U?

$\alpha_{eff} := \frac{70}{100} \cdot (\alpha - 1) + 1$ $x_p := \frac{3}{100}$ $x_f := \frac{0.724}{100}$ $\alpha_{eff} = 1.068$

$Np := \frac{ln\left[\frac{x_p \cdot \left(1 - x_f\right)}{x_f \left(1 - x_p\right)}\right]}{ln\left(\alpha_{eff}\right)}$ $Np = 22.073$ but Np must be integer, hence: $Np = 22$