(4.8) The interior of the earth is assumed to be built up of a solid core (radius 1371 km) followed by a molten core (radius 3471 km) and a molten mantle (radius 6354 km) covered by a 17 km thick crust. One assumes that 2% by weight of the molten mantle and crust is potassium; the mantle density is assumed to be 6000 kg m^{-3} and that of the crust 3300 kg m^{-3}. What energy outflow will the radioactive decay of this element cause at the earths surface? The decay scheme of ^{40}K is given in eqn. (12.16). For the EC branch Q = 1.505 Mev, for the beta-branch 1.314 MeV. Compare this energy output to the solar energy influx to the earth of 3.2×10^{24} J y^{-1}.

$Sunflux := 3.2 \cdot 10^{24} \cdot joule \cdot yr^{-1}$ $\qquad N_A := 6.022137 \cdot 10^{23} \cdot mole^{-1}$

$r_y := 6354 \cdot 10^3 \cdot m$ $\qquad r_i := 3471 \cdot 10^3 \cdot m$ $\qquad r_{cr} := r_y + 17000 \cdot m$

$d := 5000 \cdot kg \cdot m^{-3}$ $\qquad d_{cr} := 3300 \cdot kg \cdot m^{-3}$

$V_{mantle} := \frac{4}{3} \cdot \pi \cdot \left(r_y^3 - r_i^3 \right)$ $\qquad M_{mantle} := V_{mantle} \cdot d$ $\qquad M_{mantle} = 4.497 \cdot 10^{24} \cdot \mathrm{kg}$

$V_{crust} := \frac{4}{3} \cdot \pi \cdot \left(r_{cr}^3 - r_y^3 \right)$ $\qquad M_{crust} := V_{crust} \cdot d_{cr}$ $\qquad M_{crust} = 2.854 \cdot 10^{22} \cdot \mathrm{kg}$

$M_K := \left(M_{mantle} + M_{crust} \right) \cdot \frac{2}{100}$ $\qquad M_{K40} := M_K \cdot \frac{0.0117}{100}$ $\qquad M_{K40} = 1.059 \cdot 10^{19} \cdot \mathrm{kg}$

$M_{wK40} := 40 \cdot gm \cdot mole^{-1}$

$N_{K40} := \frac{M_{K40}}{M_{wK40}} \cdot N_A$ $\qquad t_{half} := 1.28 \cdot 10^9 \cdot yr$ $\qquad \lambda := \frac{ln(2)}{t_{half}}$ $\qquad \lambda = 1.716 \cdot 10^{-17} \cdot \mathrm{sec}^{-1}$

$R_{K40} := \lambda \cdot N_{K40}$ $\qquad R_{K40} = 2.736 \cdot 10^{27} \cdot \mathrm{sec}^{-1}$

If we include the energy of escaping neutrinos, i.e. use total decay energies, the energy production rate is:

$E_{decay} := (0.1067 \cdot 1.505 + 0.8933 \cdot 1.312) \cdot 10^6 \cdot 1.6021773 \cdot 10^{-19} \cdot joule$

$E_1 := R_{K40} \cdot E_{decay}$ $\qquad E_1 = 1.843 \cdot 10^{22} \cdot \frac{joule}{yr}$

$Y := 4 \cdot \pi \cdot r_{cr}^2$ $\qquad Y = 5.101 \cdot 10^{14} \cdot \mathrm{m}^2$ $\qquad \frac{E_1}{Y} = 1.145 \cdot watt \cdot m^{-2}$ $\qquad \frac{E_1}{Sunflux} = 0.5760 \cdot \%$

If we consider energy loss by escaping neutrinos, i.e. use average β-energies from Table of Isotopes, we obtain:

$Q_{eff} := (.1561 + .455 + 0.00062) \cdot 10^6 \cdot 1.6021773 \cdot 10^{-19} \cdot joule$

$E_2 := R_{K40} \cdot Q_{eff}$ $\qquad E_2 = 8.462 \cdot 10^{21} \cdot \frac{joule}{yr}$ $\qquad \frac{E_2}{Y} = 0.526 \cdot \mathrm{m}^{-2} \cdot watt$

Solar influx 1.94 cal/cm^2 min

$k_{solar} := 1.94 \cdot cal \cdot cm^{-2} \cdot min^{-1}$ $\qquad k_{solar} = 1.354 \cdot 10^3 \cdot \mathrm{m}^{-2} \cdot watt$

$A := \pi \cdot r_{cr}^2 \cdot k_{solar}$ $\qquad A = 1.726 \cdot 10^{17} \cdot \mathrm{sec}^{-1} \cdot joule$ $\qquad P_{heat} := A \cdot 0.58$

$r_{cr} = 6.371 \cdot 10^6 \cdot \mathrm{m}$ $\qquad A \cdot 0.58 = 1.001 \cdot 10^{17} \cdot \mathrm{sec}^{-1} \cdot joule$ $\qquad$ (J/y becomes heat)

$P_{heat} = 3.16 \cdot 10^{24} \cdot \frac{joule}{yr}$