(4.10) (a) Prove the correctness of eqn. (4.20) by using Newton's laws of motion. (b) Prove the correctness of eqn. (4.34).

(a): Eqn. (4.20) is: $m = m^o + (1/2)\, m^o\, v^2 / \boldsymbol{c}^2$

First according to Einstein: $E = m^*\boldsymbol{c}^2$(eqn. (3.3)), which can be used to convert the masses to energies as follows:

Multiply equation (4.20) by $\boldsymbol{c}^2$, then;

$m^*\boldsymbol{c}^2 = m^{o*}\boldsymbol{c}^2 + (1/2)m^o v^2$, or $E_{tot} = E_{mass} + E_{kin}$; hence $E_{kin} = (1/2)m^o v^2$, which agrees with

Newton's laws of motion. **Q.E.D.**

(b): Eqn. (4.34) is: $E_d = E_\gamma^2/(2m_d\boldsymbol{c}^2)$

The law of constant impulse: $m_\gamma{}^*\boldsymbol{c} + m_d{}^*v_d = 0$ (because nucleus is initially at rest, and γ has velocity $\boldsymbol{c}$)

which can be rearranged to: $m_\gamma{}^*\boldsymbol{c} = -m_d{}^*v_d$

then square this equation: $m_\gamma{}^*m_\gamma{}^*\boldsymbol{c}^2 = m_d{}^*m_d{}^*v_d{}^2$; divide by 2 and substitute $m_\gamma{}^*\boldsymbol{c}^2 = E_\gamma$;

$m_\gamma{}^*E_\gamma/2 = m_d{}^*(1/2)m_d{}^*v_d{}^2 = m_d{}^*E_d$; if we assume that v_d is much smaller than $\boldsymbol{c}$.

but the γ-quantum has no rest mass and thus $m_\gamma = E_\gamma/\boldsymbol{c}^2$

by introduction of this equation we obtain: $E_\gamma^2/(2\boldsymbol{c}^2) = m_d{}^*E_d$, which can be rearranged to yield

$E_d = E_\gamma^2/(2^*m_d{}^*\boldsymbol{c}^2)$; **Q.E.D.**