(5.7) A uranium mineral was found to contain the lead isotopes ^{204}Pb, ^{206}Pb, and ^{207}Pb in the ratio 1:1087:388, as determined with a mass spectrometer. Estimate the age of the mineral.
^{204}Pb is stable and not of a product of the decay series. A normal lead contains 1.4% of this isotope, 24.1% of ^{206}Pb, 22.1 % of ^{207}Pb, and 52.4% of ^{208}Pb.

Put $\quad m_{204} := 1 \quad m_{206} := \frac{1087}{1} \cdot m_{204} \quad m_{207} := \frac{388}{1087} \cdot m_{206} \quad \frac{m_{206}}{m_{207}} = 2.8015$

$m_{204} = 1.0000 \quad m_{206} = 1.0870 \cdot 10^3 \quad m_{207} = 388.0000$

Correct for natural lead: $\quad m_{nat204} := 1 \quad m_{nat206} := \frac{24.1}{1.4} \quad m_{nat207} := \frac{22.1}{1.4}$

$m_{nat206} = 17.2143 \quad m_{nat207} = 15.7857$

$m_{c206} := m_{206} - m_{nat206} \quad m_{c207} := m_{207} - m_{nat207} \quad \frac{m_{nat206}}{m_{nat207}} = 1.0905$

$m_{c206} = 1.0698 \cdot 10^3 \quad m_{c207} = 372.2143$

$ratio := \frac{m_{c206}}{m_{c207}} \quad ratio = 2.8741$

Now we need the decay constants for ^{235}U and ^{238}U. The corresponding half-lives are given in Figure 5.1 on p. 101.

$t_{235} := 7.04 \cdot 10^8 \cdot yr \quad \lambda_{235} := \frac{ln(2)}{t_{235}}$

$t_{238} := 4.468 \cdot 10^9 \cdot yr \quad \lambda_{238} := \frac{ln(2)}{t_{238}}$

Then we will use eqn. (5.11) to calculate the age of the mineral:

$$\frac{1}{ratio} = \frac{1}{138} \cdot \frac{e^{\lambda_{235} \cdot age} - 1}{e^{\lambda_{238} \cdot age} - 1}$$

Simplify first:

$$\frac{e^{\lambda_{235} \cdot age} - 1}{e^{\lambda_{238} \cdot age} - 1} = 48.0148$$

This equation has to be solved numerically, e.g. as follows:

$$f(age) := \frac{e^{\lambda_{235} \cdot age} - 1}{e^{\lambda_{238} \cdot age} - 1} - 48.0148$$

By locating a zero value of $f(age)$ we have determined *age*

Start by guessing a value of *age*. Then find the root by iteration: $\quad age := 10^9 \cdot yr$

$root(f(age), age) = 3.7010 \cdot 10^9 \cdot yr$

(note: the procedure can also be performed graphically by plotting $f(age)$ against *age* and locating a zero crossover point)