(5.8) The heat flow from the earth's crust is 0.060 J m^{-2} s^{-1}. The mean thichness of the crust is 17 km and the earth's radius is 6371 km. The average concentration of uranium, thorium, and potassium in granite is estimated to be 4 ppm (by weight), 18 ppm, and 3.6%, respectively. Assuming that 7% of the crust is made up of granite (feldspar + quartz, density 2.6 g cm^{-3}), what will the heat flow at the earth's surface be from each of these elements? Assume β-heat as 1/3 E_{max}; for α-decay assume $E_\alpha = Q_\alpha$. Discuss the results.

$N_A := 6.022 \cdot 10^{23} \cdot mole^{-1}$ $q_e := 1.602177 \cdot 10^{-19} \cdot coul$ $r_y := 6371 \cdot km$ $crust := 17 \cdot km$

$r_i := r_y - crust$ $d_{granite} := 2.6 \cdot gm \cdot cm^{-3}$ $v_{frac} := 7 \cdot \%$

$m_{granite} := \frac{4}{3} \cdot \pi \cdot \left[r_y^3 - (r_i)^3 \right] \cdot v_{frac} \cdot d_{granite}$ $m_{granite} = 1.5739 \cdot 10^{21} \cdot kg$

$m_U := m_{granite} \cdot 4 \cdot 10^{-6}$ $m_U = 6.2957 \cdot 10^{15} \cdot kg$

$m_{Th} := m_{granite} \cdot 18 \cdot 10^{-6}$ $m_{Th} = 2.8331 \cdot 10^{16} \cdot kg$

$m_K := m_{granite} \cdot 3.6 \cdot 10^{-2}$ $m_K = 5.6662 \cdot 10^{19} \cdot kg$ $f_{K40} := 0.0117 \cdot \%$

$m_{U235} := m_U \cdot 0.72 \cdot 10^{-2}$ $m_{U235} = 4.5329 \cdot 10^{13} \cdot kg$

$m_{U238} := m_U \cdot 99.2745 \cdot 10^{-2}$ $m_{U238} = 6.2501 \cdot 10^{15} \cdot kg$

$m_{Th232} := m_{Th}$ $m_{Th232} = 2.8331 \cdot 10^{16} \cdot kg$

$m_{K40} := m_K \cdot f_{K40}$ $m_{K40} = 6.6294 \cdot 10^{15} \cdot kg$

$t_{h235} := 7.038 \cdot 10^8 \cdot yr$ $M_{235} := 235 \cdot gm \cdot mole^{-1}$ $\lambda_{235} := \frac{ln(2)}{t_{h235}}$

$t_{h238} := 4.468 \cdot 10^9 \cdot yr$ $M_{238} := 238 \cdot gm \cdot mole^{-1}$ $\lambda_{238} := \frac{ln(2)}{t_{h238}}$

$t_{h232} := 1.405 \cdot 10^{10} \cdot yr$ $M_{232} := 232 \cdot gm \cdot mole^{-1}$ $\lambda_{232} := \frac{ln(2)}{t_{h232}}$

$t_{h40} := 1.28 \cdot 10^9 \cdot yr$ $M_{40} := 40 \cdot gm \cdot mole^{-1}$ $\lambda_{40} := \frac{ln(2)}{t_{h40}}$

$R_{235} := \lambda_{235} \cdot \frac{m_{U235}}{M_{235}} \cdot N_A$ $R_{238} := \lambda_{238} \cdot \frac{m_{U238}}{M_{238}} \cdot N_A$

$R_{232} := \lambda_{232} \cdot \frac{m_{Th232}}{M_{232}} \cdot N_A$ $R_{40} := \lambda_{40} \cdot \frac{m_{K40}}{M_{40}} \cdot N_A$

$Q_{235} := \left(4.6785 + \frac{1}{3} \cdot 0.3 + 5.1482 + \frac{1}{3} \cdot 0.04 + 6.14649 + 5.9792 + 6.9461 + 7.5264 + \frac{1}{3} \cdot 1.4 + 6.7511 + \frac{1}{3} \cdot 1.4 \right) \cdot 1.6021773 \cdot 10^{-13} \cdot joule$

$Q_{235} = 7.0853 \cdot 10^{-12} \cdot joule$

$P_{235} := R_{235} \cdot Q_{235}$ $P_{235} = 2.5686 \cdot 10^{10} \cdot watt$

$Q_{238} := \left[\begin{array}{l} 4.274 + 4.8585 + 4.7711 + 4.87067 + 5.5904 + 6.11473 + 7.83353 + 5.4075 \ ... \\ + \frac{1}{3} \cdot (0.2 + 2.3 + 0.7 + 1.5 + 0.02 + 1.2) \end{array} \right] \cdot 1.602 \cdot 10^{-13} \cdot joule$

$Q_{238} = 7.3201 \cdot 10^{-12} \cdot joule$

$P_{238} := R_{238} \cdot Q_{238}$ $P_{238} = 5.6909 \cdot 10^{11} \cdot watt$

$Q_{232} := \left[4.083 + 5.52014 + 5.78890 + 6.40472 + 6.9064 + 8.95336 + \frac{1}{3} \cdot (0.04 + 1.2 + 0.3 + 2.3) \right] \cdot 1.6021773 \cdot 10^{-13} \cdot joule$

$Q_{232} = 6.2383 \cdot 10^{-12} \cdot joule$

$P_{232} := R_{232} \cdot Q_{232}$ $P_{232} = 7.1719 \cdot 10^{11} \cdot watt$

$Q_{40} := \frac{1.3}{3} \cdot 1.6021773 \cdot 10^{-13} \cdot joule$

$P_{40} := R_{40} \cdot Q_{40}$ $P_{40} = 1.1891 \cdot 10^{11} \cdot watt$

$Area := 4 \cdot \pi \cdot r_y^2$ $P_{tot} := P_{235} + P_{238} + P_{232} + P_{40}$ $\frac{P_{tot}}{Area} = 2.8053 \cdot 10^{-3} \cdot m^{-2} \cdot watt$

$Area = 5.1006 \cdot 10^{14} \cdot m^2$

$\frac{P_{235}}{Area} = 5.0357 \cdot 10^{-5} \cdot m^{-2} \cdot watt$

$\frac{P_{238}}{Area} = 1.1157 \cdot 10^{-3} \cdot m^{-2} \cdot watt$

$\frac{P_{232}}{Area} = 0.0014 \cdot m^{-2} \cdot watt$

$\frac{P_{40}}{Area} = 0.00023 \cdot m^{-2} \cdot watt$

$\frac{P_{235} + P_{238}}{Area} = 0.0012 \cdot \frac{watt}{m^2}$