(6.8) The E_{max} of ^{32}P β-particles is 1.71 MeV. To what electron velocity does this correspond?

Begin by defining the rest mass of an electron and the velocity of light (data from Appendix III.):

$M_e := 5.485799 \cdot 10^{-4}$ u $\qquad c := 299792458 \cdot \frac{m}{sec}$

The given maximum β-energy is:

$E_{max} := 1.71$ MeV

Eqn. (4.21): $m = m^o + E_{kin}/\boldsymbol{c}^2$

Eqn. (4.19): $m = m^o (1 - v^2/\boldsymbol{c}^2)^{-1/2}$

By combining these equations, m can be eliminated. After some simple algebra we obtain the following equation, where β stands for $v/\boldsymbol{c}$

$$\beta = \sqrt{1 - \left[\frac{1}{\left(\frac{E_{max}}{m_e \cdot c^2} + 1\right)}\right]^2}$$

To make the numerical calculation simpler, we can use the mass-energy equivalent of 931.5 MeV/u and replace m_e (in kg) by M_e (in u). This results in:

$$\beta := \sqrt{1 - \left[\frac{1}{\left(\frac{E_{max}}{931.5 \cdot M_e} + 1\right)}\right]^2}$$

$\beta = 0.9732$ (97.3% of $\boldsymbol{c}$)

But: $v := \beta \cdot c$

and thus: $v = 2.917 \cdot 10^8 \cdot m \cdot sec^{-1}$