(8.3) Plutonium in an urine sample is soaked into a photographic emulsion so that the emulsion increases its volume by 20%. The 12 μm thick emulsion is dried to original thickness and then left in darkness for 24 h. After development, α-tracks are counted and an average of 2356 tracks cm^{-2} found. If the plutonium consists of 67% ^{239}Pu and 33% ^{240}Pu, what was the plutonium concentration in the urine?

First the needed definitions of units, etc.

$Bq := sec^{-1}$ $M := mole \cdot liter^{-1}$ $\mu M := 10^{-6} \cdot M$ $\mu m := 10^{-6} \cdot m$ $\mu g := 10^{-6} \cdot gm$

$N_A := 6.0221367 \cdot \frac{10^{23}}{mole}$ $M_{239} := 239 \cdot \frac{gm}{mole}$ $M_{240} := 240 \cdot \frac{gm}{mole}$

Then the calculations:

$Volume := \frac{20}{100} \cdot 12 \cdot \mu m \cdot 1 \cdot cm^2$ $Volume = 2.4 \cdot 10^{-4} \cdot mL$ urine/cm^2 plate

$A := \frac{2365}{24 \cdot hr}$ $A = 0.027 \cdot Bq$ $S := \frac{A}{Volume}$ $S = 1.141 \cdot 10^{5} \cdot \frac{Bq}{liter}$ urine

$t_{239} := 2.411 \cdot 10^4 \cdot yr$ $\lambda_{239} := \frac{ln(2)}{t_{239}}$ $\lambda_{239} = 9.11 \cdot 10^{-13} \cdot \sec^{-1}$

$t_{240} := 6550 \cdot yr$ $\lambda_{240} := \frac{ln(2)}{t_{240}}$ $\lambda_{240} = 3.353 \cdot 10^{-12} \cdot \sec^{-1}$

$Mw := M_{239} \cdot \frac{67}{100} + M_{240} \cdot \frac{33}{100}$ $Mw = 239.33 \cdot \frac{gm}{mole}$

$N_{Pu} := \frac{S}{\lambda_{239} \cdot \frac{67}{100} + \lambda_{240} \cdot \frac{33}{100}}$ Pu atoms/liter

$C_{Pu} := \frac{N_{Pu}}{N_A}$ $c_{Pu} := C_{Pu} \cdot Mw$ g/liter of urine

$C_{Pu} = 0.11 \cdot \mu M$ $c_{Pu} = 26.4 \cdot \frac{\mu g}{liter}$ urine

The Pu concentration in the urine sample was 0.11 μM or 26.4 μg/liter.