(9.1) The blood volume of a patient is to be determined by means of $^{32}P$. For this purpose 15.0 ml of blood is withdrawn from the patient and mixed wit a very small volume of $Na_2H^{32}PO_4$ of high specific activity. In 1 h the erythrocytes (red blood cells) take up all the $^{32}P$; 1 ml is found to have an activity of 216000 cpm in the detector system used. Exactly 5.00 ml of this tagged sample is injected into the patient, and 30 min later a new sample is withdrawn; 10 ml of this gives 2300 cpm. Calculate the blood volume.

Some definition of units:

$$Bq := sec^{-1} \qquad cpm := min^{-1}$$

First of all check if any corrections for radioactive decay are necessary:

$$t_{half} := 14.3 \cdot day \qquad Rfac := 2^{\frac{30 \cdot min}{t_{half}}} \qquad Rfac = 1.001$$

Hence 30 minutes make no practical difference and no corrections are really necessary. However, we will do it here anyway in order to show the method

Given in the text are:

$$S_0 := 216000 \cdot cpm \cdot mL^{-1} \qquad v_0 := 5 \cdot mL \qquad A := 2300 \cdot cpm \qquad v_A := 10 \cdot mL$$

Now we correct for the decay:

$$A_{corr} := A \cdot Rfac$$

Next; calculate the specific activity of the blood:

$$S := \frac{A_{corr}}{v_A} \qquad S = 3.837 \cdot \frac{Bq}{mL}$$

Use: $v_0 * S_0 = (v_0 + V * S)$(a simple mass balance on activity) to compute the blood volume (neglect $v_0$ in comparison with $V$)

$$V := \frac{v_0 \cdot S_0}{S} \qquad V = 4.691 \cdot 10^{-3} \cdot m^3 \qquad V = 4.7 \cdot liter$$