

(11.2) In the hydrogen atom the K-electron radius is assumed to be $0.529 \cdot 10^{-10}$ m (the Bohr radius). (a) Calculate the orbital velocity of the electron assuming its mass to be m_e . (b) How much larger is its real mass because of the velocity? Does this affect the calculations in (a)?

(a)

$$r := 0.529 \cdot 10^{-10} \cdot m \quad m_e := 9.109390 \cdot 10^{-31} \cdot kg \quad q_e := 1.6021773 \cdot 10^{-19} \cdot coul$$

$$\epsilon_0 := 8.8541878 \cdot 10^{-12} \cdot \frac{amp \cdot sec}{volt \cdot m} \quad \epsilon_0 = 8.854 \cdot 10^{-12} \cdot kg^{-1} \cdot m^{-3} \cdot sec^2 \cdot coul^2$$

$$F := \left(\frac{q_e}{r} \right)^2 \cdot \frac{1}{4 \cdot \pi \cdot \epsilon_0} \quad \text{From physics.} \quad F = 8.244 \cdot 10^{-8} \cdot kg \cdot m \cdot sec^{-2}$$

$$v := \sqrt{\frac{r \cdot F}{m_e}} \quad \text{From mechanics} \quad v = 2.19 \cdot 10^6 \cdot m \cdot sec^{-1}$$

(b)

$$c := 2.99792458 \cdot 10^8 \cdot \frac{m}{sec} \quad \beta := \frac{v}{c} \quad \beta = 7.299 \cdot 10^{-3}$$

$$m_r := \frac{m_e}{\sqrt{1 - \beta^2}} \quad \text{Eqn.(4.19)}$$

$$\frac{m_r - m_e}{m_e} = 2.664 \cdot 10^{-3} \cdot \%$$

Thus the mass has increased by 0.003%

$$v_r := \sqrt{\frac{r \cdot F}{m_r}} \quad v_r = 2.19 \cdot 10^6 \cdot m \cdot sec^{-1}$$

$$\Delta v := \frac{v - v_r}{v} \quad \Delta v = 1.332 \cdot 10^{-3} \cdot \%$$

The velocity is practically the same.