(12.2) Calculate the distance of closest approach for 5 MeV α-particles to a gold target.

Constants and units:

$fm := 10^{-15} \cdot m$ $\quad$ $MeV := 1.60217733 \cdot 10^{-13} \cdot joule$

$\varepsilon_0 := 8.8541878 \cdot 10^{-12} \cdot \frac{coul}{volt \cdot m}$ $\quad$ $q_e := 1.6021773 \cdot 10^{-19} \cdot coul$ $\quad$ $u_n := 1.660540 \cdot 10^{-27} \cdot kg$

Given data from the text:

$Z_\alpha := 2$ $\quad$ $Z_{Au} := 79$ $\quad$ $A_{Au} := 197$

Calculations:

$k := \frac{1}{4 \cdot \pi \cdot \varepsilon_0}$ $\quad$ $k$ in eqn. (12.12)

$m_\alpha := \left(4.002603 - 2 \cdot 5.4857990 \cdot 10^{-4}\right) \cdot u_n$ $\quad$ mass of He without two electrons

$m_{Au} := 196.97 \cdot u_n$

$E_\alpha := 5 \cdot MeV$ $\quad$ $E_\alpha = 8.011 \cdot 10^{-13} \cdot joule$

$E_{CM} := \frac{E_\alpha \cdot m_{Au}}{\left(m_\alpha + m_{Au}\right)}$ $\quad$ Center-of-mass energy which can be derived from (4.3), (4.4), assuming $E_{kin} = m*v^2/2$.

$r_c := \frac{k \cdot Z_{Au} \cdot Z_\alpha \cdot q_e^2}{E_{CM}}$ $\quad$ Eqn. (12.13) solved for $r_c$

$r_c = 4.6 \cdot 10^{-14} \cdot m$ $\quad$ $r_c = 46.4 \cdot fm$

A simple derivation of the CM-energy equation for a projectile p and a target A is as follows:

Begin with the conservation of linear momentum:

$$m_p \cdot v_p = \left(m_p + m_A\right) \cdot v_{CM}$$

Then square this equation and divide both sides by 2 to convert momentum into kinetic energy:

$$\frac{\left(m_p \cdot v_p\right)^2}{2} = \frac{\left[\left(m_p + m_A\right) \cdot v_{CM}\right]^2}{2}$$

$$m_p \cdot E_p = \left(m_p + m_A\right) \cdot E_{CM}$$

$$E_{CM} = E_p \cdot \frac{m_p}{m_p + m_a}$$