

(12.4) In Rutherford scattering on a silver foil using α -particles from a thin-walled radon tube, the following data were observed: $d\sigma/d\Omega = 22(\theta 150^\circ)$, $47(105^\circ)$, $320(60^\circ)$, $5260(30^\circ)$, $105400(15^\circ)$ barns per steradian. Calculate the energy of the incident α -particles.

Use eqn (12.17) $d\sigma/d\Omega = (k \cdot Z_1 \cdot Z_2 \cdot e^2 / (2 \cdot m_{red} \cdot (v_1^0)^2))^2 / \sin^4(\theta/2)$ and then $E_{kin} = m \cdot v^2/2$:

Define some useful constants and units:

$$\epsilon_0 := 8.8541878 \cdot 10^{-12} \frac{\text{coul}}{\text{volt} \cdot \text{m}} \quad q_e := 1.6021773 \cdot 10^{-19} \cdot \text{coul} \quad u_n := 1.660540 \cdot 10^{-27} \cdot \text{kg}$$

$$\text{barn} := 10^{-28} \cdot \text{m}^2 \quad M_{Ag} := 107.87 \cdot u_n$$

Calculations:

$$k := \frac{1}{4 \cdot \pi \cdot \epsilon_0} \quad k \text{ in eqn. (12.12)}$$

$$M_\alpha := (4.002603 - 2 \cdot 5.4857990 \cdot 10^{-4}) \cdot u_n \quad (\alpha = \text{He}^{2+})$$

$$m_{red} := \frac{1}{\left(\frac{1}{M_{Ag}} + \frac{1}{M_\alpha}\right)} \quad Z_1 := 2 \quad Z_2 := 47 \quad m_{red} = 6.407 \cdot 10^{-27} \cdot \text{kg}$$

$$k = 8.988 \cdot 10^9 \cdot \text{kg} \cdot \text{m}^3 \cdot \text{sec}^{-2} \cdot \text{coul}^{-2}$$

$i := 1 \dots 5$

$$d\sigma d\Omega_i := \theta_i := \theta_{r_i} := \theta_i \cdot \frac{\pi}{180}$$

22
47
320
5260
105400

150
105
60
30
15

$6.219 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$
$6.263 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$
$6.152 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$
$5.902 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$
$5.532 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$

$$v_{1_i} := \sqrt{\frac{k \cdot Z_1 \cdot Z_2 \cdot q_e^2}{2 \cdot m_{red} \cdot \sqrt{\left(\text{barn} \cdot d\sigma d\Omega_i\right)} \cdot \sin\left(\frac{\theta_{r_i}}{2}\right) \cdot \sin\left(\frac{\theta_{r_i}}{2}\right)}}$$

$$v_{avg} := \frac{\sum_{i=1}^5 v_{1_i}}{5}$$

$$v_{avg} = 6.013 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$$

$$E_{avg} := \frac{1}{2} \cdot M_\alpha \cdot v_{avg}^2$$

$$E_{avg} = 1.201 \cdot 10^{-13} \cdot \text{joule}$$

$$= \frac{1.201 \cdot 10^{-13} \cdot \text{coul}}{q_e} = 7.5 \cdot 10^5 \quad \text{eV}$$

