

(13.1) In a small linear accelerator containing 30 stages, He^{2+} ions are accelerated by a 150 kV, 100 MHz RF source. The ions are used to bombard a metal target to induce a specific reaction. (a) What is the proper length of the last drift tube? (b) What is the maximum projectile energy achieved? (c) What is the heaviest target in which a nuclear transformation can be induced (no tunneling)?

Basic definitions of units and constants:

$$\begin{aligned} q_e &:= 1.6021773 \cdot 10^{-19} \cdot \text{coul} & c_{\text{light}} &:= 299792458 \cdot \text{m} \cdot \text{sec}^{-1} & \text{MeV} &:= 1.60217733 \cdot 10^{-13} \cdot \text{joule} \\ \text{amu} &:= 1.6605402 \cdot 10^{-27} \cdot \text{kg} & N_A &:= 6.0221367 \cdot 10^{23} \cdot \text{mole}^{-1} & \text{kV} &:= 1000 \cdot \text{volt} \\ M_e &:= 5.485799 \cdot 10^{-4} \cdot \frac{\text{gm}}{\text{mole}} & M_{\text{He}} &:= 4.002603 \cdot \frac{\text{gm}}{\text{mole}} \end{aligned}$$

Data for the accelerator:

$$E_{\text{acc}} := 150 \cdot \text{kV} \quad n_{\text{stages}} := 30 \quad z := 2 \quad f_{\text{RF}} := 100 \cdot \text{MHz}$$

$$\lambda := \frac{c_{\text{light}}}{f_{\text{RF}}} \quad m_0 := \frac{M_{\text{He}} - z \cdot M_e}{N_A} \quad m_0 = 6.645 \cdot 10^{-27} \cdot \text{kg}$$

Calculations:

$$(a) \quad k_x := \frac{q_e \cdot z \cdot E_{\text{acc}}}{m_0 \cdot c_{\text{light}}^2} \quad L_{30} := \frac{\lambda}{2} \cdot \sqrt{1 - (n_{\text{stages}} \cdot k_x + 1)^{-2}} \quad L_{30} = 10.4 \cdot \text{cm}$$

$$(b) \quad E_{\text{proj}} := n_{\text{stages}} \cdot E_{\text{acc}} \cdot z \cdot q_e \quad E_{\text{proj}} = 9 \cdot \text{MeV}$$

$$(c) \quad A_1 := 4 \quad Z_1 := 2 \quad i := 1..8 \quad \text{Make a Table of possible targets and test each value:}$$

$$A_i := \begin{array}{|c|} \hline 9 \\ \hline 19 \\ \hline 23 \\ \hline 27 \\ \hline 31 \\ \hline 38 \\ \hline 45 \\ \hline 55 \\ \hline \end{array} \quad Z_i := \begin{array}{|c|} \hline 4 \\ \hline 9 \\ \hline 11 \\ \hline 13 \\ \hline 15 \\ \hline 18 \\ \hline 21 \\ \hline 25 \\ \hline \end{array} \quad \text{eqn.(12.14)} \quad E_i := \left[1.109 \cdot (A_1 + A_i) \cdot \frac{Z_1 \cdot Z_i}{A_i \left[A_1^{\frac{1}{3}} + (A_i)^{\frac{1}{3}} \right]} - 9 \right] \cdot \text{MeV}$$

Heaviest target about ${}^{38}\text{Ar}$ (Z for A from Table above)

