(13.3) In order to propel a space vehicle to high velocities after its exit from the earth's gravitational pull, "ion rockets" might be used. These can be considered as simple accelerators for charged particles. The electroneutrality of the vehicle is conserved through emission of electrons from a hot filament. The gain in linear velocity Δv is given by the "rocket formula": $\Delta v = v_e \ln(m_{Ro}/m_R)$, where v_e is the exhaust velocity and m_{Ro} and m_R are the initial and final mass of the vehicle respectively. (a) Calculate the propelling power for a rocket, which emits 10 keV protons at a current of 1 A. (b) What is the final velocity gain of such vehicle with 2000 kg initial mass after 1 year's operating time? (c) For the same net available power and operating time, as in (b), calculate the final velocity gain of a 2000 kg vehicle which emits 10 keV Cs^+ ions at a current of 1 A.

First the usual basic definitions of units and constants:

$q_e := 1.6021773 \cdot 10^{-19} \cdot coul$ $\qquad N_A := 6.022137 \cdot 10^{23} \cdot mole^{-1}$ $\qquad eV := 1.6021773 \cdot 10^{-19} \cdot joule$

The initial mass of the vehicle is: $m_{Ro} := 2000 \cdot kg$

(a) $P_{prop} := 1 \cdot 10 \cdot 10^3 \cdot watt$ Basic physics: Current*Voltage=Power $\qquad P_{prop} = 10 \cdot kW$

(b) $M_p := 1.00727647 \cdot gm \cdot mole^{-1}$ $\qquad I_p := 1 \cdot amp$ $\qquad E_p := 10 \cdot 10^3 \cdot eV$

$m_p := \frac{M_p}{N_A}$ $\qquad v_e := \sqrt{2 \cdot \frac{E_p}{m_p}}$ $\qquad \Delta m\Delta t := \frac{I_p}{q_e} \cdot m_p$ $\qquad t := 365 \cdot 24 \cdot 60 \cdot 60 \cdot sec$

$\Delta m := \Delta m\Delta t \cdot t$ $\qquad m_R := m_{Ro} - \Delta m$ $\qquad \Delta v := v_e \cdot ln\left(\frac{m_{Ro}}{m_R}\right)$ $\qquad \Delta v = 228 \cdot m \cdot sec^{-1}$

(c) Because P_{prop} is the same, I is the same

$M_{Cs} := 132.9 \cdot gm \cdot mole^{-1}$ $\qquad I_{Cs} := 1 \cdot amp$ $\qquad E_{Cs} := 10 \cdot 10^3 \cdot eV$

$m_{Cs} := \frac{M_{Cs}}{N_A}$ $\qquad v_e := \sqrt{2 \cdot \frac{E_{Cs}}{m_{Cs}}}$ $\qquad \Delta m\Delta t := \frac{I_{Cs}}{q_e} \cdot m_{Cs}$ $\qquad t := 365 \cdot 24 \cdot 60 \cdot 60 \cdot sec$

$\Delta m := \Delta m\Delta t \cdot t$ $\qquad m_R := m_{Ro} - \Delta m$ $\qquad \Delta v := v_e \cdot ln\left(\frac{m_{Ro}}{m_R}\right)$ $\qquad \Delta v = 2646 \cdot m \cdot sec^{-1}$