(19.10) Our solar system is considered to be 4.5 billion years old. What was the ^{235}U percentage in natural uranium when the solar system was formed?

Known data and constants:

$N_A := 6.022137 \cdot 10^{23} \cdot mole^{-1}$ $M_{235} := 235 \cdot gm \cdot mole^{-1}$ $M_{238} := 238 \cdot gm \cdot mole^{-1}$

$t_{h235} := 7.038 \cdot 10^{8} \cdot yr$ $\lambda_{235} := \frac{ln(2)}{t_{h235}}$ $x_{235} := 0.72 \cdot \%$

$t_{h238} := 4.468 \cdot 10^{9} \cdot yr$ $\lambda_{238} := \frac{ln(2)}{t_{h238}}$ $x_{238} := 1 - x_{235}$

Data given in the text:

$t_{age} := 4.5 \cdot 10^{9} \cdot yr$

Assume 1 kg uranium for simplicity: $m_{tot} := 1 \cdot kg$

Calculations:

$M_U := x_{235} \cdot M_{235} + x_{238} \cdot M_{238}$

$N_{235} := \frac{m_{tot}}{M_U} \cdot N_A \cdot x_{235}$ $N_{238} := \frac{m_{tot}}{M_U} \cdot N_A \cdot x_{238}$

$N_{0235} := N_{235} \cdot exp\left(\lambda_{235} \cdot t_{age}\right)$ $m_{old235} := \frac{N_{0235}}{N_A} \cdot M_{235}$

$N_{0238} := N_{238} \cdot exp\left(\lambda_{238} \cdot t_{age}\right)$ $m_{old238} := \frac{N_{0238}}{N_A} \cdot M_{238}$

$m_{old} := m_{old235} + m_{old238}$ $Fraction_{235} := \frac{m_{old235}}{m_{old}}$ $Fraction_{235} = 0.231$

or

$Fraction_{235} = 23.1 \cdot \%$