(5.8) The interior of the earth is assumed to be built up of a solid core (radius 1371 km) followed by a molten core (radius 3471 km) and a molten mantle (radius 6354 km) covered by a 17 km thick crust. One assumes that 2% by weight of the molten mantle and crust is potassium; the mantle density is assumed to be 6000 kg m^{-3} and that of the crust 3300 kg m^{-3}. What energy outflow will the radioactive decay of this element cause at the earths surface? The decay scheme of ^{40}K is given in eqn. (13.7). For the EC branch Q = 1.505 Mev, for the beta-branch 1.314 MeV. Compare this energy output to the solar energy influx to the earth of 3.2×10^{24} J y^{-1}.

$Sunflux := 3.2\cdot 10^{24}\cdot joule\cdot yr^{-1}$ $\qquad N_A := 6.022137\cdot 10^{23}\cdot mole^{-1}$

$r_y := 6354\cdot 10^3\cdot m$ $\qquad r_i := 3471\cdot 10^3\cdot m$ $\qquad r_{cr} := r_y + 17000\cdot m$

$d := 5000\cdot kg\cdot m^{-3}$ $\qquad d_{cr} := 3300\cdot kg\cdot m^{-3}$

$V_{mantle} := \frac{4}{3}\cdot\pi\cdot\left(r_y^3 - r_i^3\right)$ $\qquad M_{mantle} := V_{mantle}\cdot d$ $\qquad M_{mantle} = 4.497\cdot 10^{24}\ \cdot kg$

$V_{crust} := \frac{4}{3}\cdot\pi\cdot\left(r_{cr}^3 - r_y^3\right)$ $\qquad M_{crust} := V_{crust}\cdot d_{cr}$ $\qquad M_{crust} = 2.854\cdot 10^{22}\ \cdot kg$

$M_K := \left(M_{mantle} + M_{crust}\right)\cdot\frac{2}{100}$ $\qquad M_{K40} := M_K\cdot\frac{0.0117}{100}$ $\qquad M_{K40} = 1.059\cdot 10^{19}\ \cdot kg$

$M_{wK40} := 40\cdot gm\cdot mole^{-1}$

$N_{K40} := \frac{M_{K40}}{M_{wK40}}\cdot N_A$ $\qquad t_{half} := 1.28\cdot 10^9\cdot yr$ $\qquad \lambda := \frac{ln(2)}{t_{half}}$ $\qquad \lambda = 1.716\cdot 10^{-17}\ \cdot sec^{-1}$

$R_{K40} := \lambda\cdot N_{K40}$ $\qquad R_{K40} = 2.736\cdot 10^{27}\ \cdot sec^{-1}$

If we include the energy of escaping neutrinos, i.e. use total decay energies, the energy production rate is:

$E_{decay} := (0.1067\cdot 1.505 + 0.8933\cdot 1.312)\cdot 10^6\cdot 1.6021773\cdot 10^{-19}\cdot joule$

$E_1 := R_{K40}\cdot E_{decay}$ $\qquad E_1 = 1.843\cdot 10^{22}\ \cdot\frac{joule}{yr}$

$Y := 4\cdot\pi\cdot r_{cr}^2$ $\qquad Y = 5.101\cdot 10^{14}\ \cdot m^2$ $\qquad \frac{E_1}{Y} = 1.145\cdot watt\cdot m^{-2}$ $\qquad \frac{E_1}{Sunflux} = 0.5760\cdot\%$

If we consider energy loss by escaping neutrinos, i.e. use average β-energies from Table of Isotopes, we obtain:

$Q_{eff} := (.1561 + .455 + 0.00062)\cdot 10^6\cdot 1.6021773\cdot 10^{-19}\cdot joule$

$E_2 := R_{K40}\cdot Q_{eff}$ $\qquad E_2 = 8.462\cdot 10^{21}\ \cdot\frac{joule}{yr}$ $\qquad \frac{E_2}{Y} = 0.526\cdot m^{-2}\ \cdot watt$

Solar influx 1.94 cal/cm^2 min

$k_{solar} := 1.94\cdot cal\cdot cm^{-2}\cdot min^{-1}$ $\qquad k_{solar} = 1.354\cdot 10^3\ \cdot m^{-2}\ \cdot watt$

$A := \pi\cdot r_{cr}^2\cdot k_{solar}$ $\qquad A = 1.726\cdot 10^{17}\ \cdot sec^{-1}\ \cdot joule$ $\qquad P_{heat} := A\cdot 0.58$

$r_{cr} = 6.371\cdot 10^6\ \cdot m$ $\qquad A\cdot 0.58 = 1.001\cdot 10^{17}\ \cdot sec^{-1}\ \cdot joule$ $\qquad$ (J/y becomes heat)

$P_{heat} = 3.16\cdot 10^{24}\ \cdot\frac{joule}{yr}$