

(9.3) Plutonium in an urine sample is soaked into a photographic emulsion so that the emulsion increases its volume by 20%. The 12 μm thick emulsion is dried to original thickness and then left in darkness for 24 h. After development, α -tracks are counted and an average of 2356 tracks cm^{-2} found. If the plutonium consists of 67% ^{239}Pu and 33% ^{240}Pu , what was the plutonium concentration in the urine?

$$\begin{aligned}
 Bq &:= \text{sec}^{-1} & M &:= \text{mole} \cdot \text{liter}^{-1} & \mu M &:= 10^{-6} \cdot M & \mu\text{m} &:= 10^{-6} \cdot \text{m} \\
 \\
 Volume &:= \frac{20}{100} \cdot 12 \cdot \mu\text{m} \cdot 1 \cdot \text{cm}^2 & & & Volume &= 2.4 \cdot 10^{-4} \cdot \text{mL} & & \text{urine/cm}^2 \text{ plate} \\
 A &:= \frac{2365}{24 \cdot \text{hr}} & A &= 0.027 \cdot Bq & S &:= \frac{A}{Volume} & S &= 1.141 \cdot 10^5 \cdot \frac{Bq}{\text{liter}} \text{ urine} \\
 \\
 t_{239} &:= 2.411 \cdot 10^4 \cdot \text{yr} & \lambda_{239} &:= \frac{\ln(2)}{t_{239}} & N_A &:= 6.0221367 \cdot \frac{10^{23}}{\text{mole}} \\
 t_{240} &:= 6550 \cdot \text{yr} & \lambda_{240} &:= \frac{\ln(2)}{t_{240}} & Mw &:= 239 \cdot \frac{\text{gm}}{\text{mole}} \cdot \frac{67}{100} + 240 \cdot \frac{\text{gm}}{\text{mole}} \cdot \frac{33}{100} \\
 \\
 N_{Pu} &:= \frac{S}{\lambda_{239} \cdot \frac{67}{100} + \lambda_{240} \cdot \frac{33}{100}} & & & & & & \text{Pu atoms/liter} \\
 \\
 C_{Pu} &:= \frac{N_{Pu}}{N_A} & c_{Pu} &:= C_{Pu} \cdot Mw & & & & \text{g/liter of urine} \\
 \\
 C_{Pu} &= 0.11 \cdot \mu M & c_{Pu} &= 2.64 \cdot 10^{-5} \cdot \frac{\text{gm}}{\text{liter}} & & & & \text{urine}
 \end{aligned}$$

The Pu concentration in the urine sample was 0.11 μM or 26.4 $\mu\text{g/liter}$.