

(13.10) A uranium mineral was found to contain the lead isotopes ^{204}Pb , ^{206}Pb , and ^{207}Pb in the ratio 1:1087:388, as determined with a mass spectrometer. Estimate the age of the mineral.

^{204}Pb is stable and not of a product of the decay series. A normal lead contains 1.4% of this isotope, 24.1% of ^{206}Pb , 22.1% of ^{207}Pb , and 52.4% of ^{208}Pb .

$$\text{Put } m_{204} := 1 \quad m_{206} := \frac{1087}{1} \cdot m_{204} \quad m_{207} := \frac{388}{1087} \cdot m_{206} \quad \frac{m_{206}}{m_{207}} = 2.8015$$

$$m_{204} = 1.0000 \quad m_{206} = 1.0870 \cdot 10^3 \quad m_{207} = 388.0000$$

$$\text{Correct for natural lead: } m_{\text{nat}204} := 1 \quad m_{\text{nat}206} := \frac{24.1}{1.4} \quad m_{\text{nat}207} := \frac{22.1}{1.4}$$

$$m_{\text{nat}206} = 17.2143 \quad m_{\text{nat}207} = 15.7857$$

$$m_{\text{c}206} := m_{206} - m_{\text{nat}206} \quad m_{\text{c}207} := m_{207} - m_{\text{nat}207} \quad \frac{m_{\text{c}206}}{m_{\text{c}207}} = 1.0905$$

$$m_{\text{c}206} = 1.0698 \cdot 10^3 \quad m_{\text{c}207} = 372.2143$$

$$\text{ratio} := \frac{m_{\text{c}206}}{m_{\text{c}207}} \quad \text{ratio} = 2.8741$$

Now we need the decay constants for ^{235}U and ^{238}U . The corresponding half-lives are given in Figure 14. ▢.

$$t_{235} := 7.04 \cdot 10^8 \cdot \text{yr} \quad \lambda_{235} := \frac{\ln(2)}{t_{235}}$$

$$t_{238} := 4.468 \cdot 10^9 \cdot \text{yr} \quad \lambda_{238} := \frac{\ln(2)}{t_{238}}$$

Then we will use eqn. (13.14) to calculate the age of the mineral:

$$\frac{1}{\text{ratio}} = \frac{1}{138} \cdot \frac{e^{\lambda_{235} \cdot \text{age}} - 1}{e^{\lambda_{238} \cdot \text{age}} - 1} \quad \text{Simplify first:}$$

$$\frac{e^{\lambda_{235} \cdot \text{age}} - 1}{e^{\lambda_{238} \cdot \text{age}} - 1} = 48.0148 \quad \text{This equation has to be solved numerically, e.g. as follows:}$$

$$f(\text{age}) := \frac{e^{\lambda_{235} \cdot \text{age}} - 1}{e^{\lambda_{238} \cdot \text{age}} - 1} - 48.0148 \quad \text{By locating a zero value of } f(\text{age}) \text{ we have determined } \text{age}$$

Start by guessing a value of *age*. Then find the root by iteration:

$$\text{age} := 10^9 \cdot \text{yr}$$

$$f(\text{age}) = 0.7010 \cdot 10^9$$

$$\text{root}(f(\text{age}), \text{age}) = 3.1010^{\cdot 10^{-7}} \cdot yf$$

(note: the procedure can also be performed graphically by plotting $f(\text{age})$ against age and locating a zero crossover point)