

(13.11) The heat flow from the earth's crust is  $0.060 \text{ J m}^{-2} \text{ s}^{-1}$ . The mean thickness of the crust is 17 km and the earth's radius is 6371 km. The average concentration of uranium, thorium, and potassium in granite is estimated to be 4 ppm (by weight), 18 ppm, and 3.6%, respectively. Assuming that 7% of the crust is made up of granite (feldspar + quartz, density  $2.6 \text{ g cm}^{-3}$ ), what will the heat flow at the earth's surface be from each of these elements? Assume  $\beta$ -heat as  $1/3 E_{\max}$ ; for  $\alpha$ -decay assume  $E_{\alpha} = Q_{\alpha}$ . Discuss the results.

$$N_A := 6.022 \cdot 10^{23} \cdot \text{mole}^{-1} \quad q_e := 1.602177 \cdot 10^{-19} \cdot \text{coul} \quad r_y := 6371 \cdot \text{km} \quad \text{crust} := 17 \cdot \text{km}$$

$$r_j := r_y - \text{crust} \quad d_{\text{granite}} := 2.6 \cdot \text{gm} \cdot \text{cm}^{-3} \quad v_{\text{frac}} := 7 \cdot \%$$

$$m_{\text{granite}} := \frac{4}{3} \cdot \pi [r_y^3 - (r_j)^3] \cdot v_{\text{frac}} \cdot d_{\text{granite}} \quad m_{\text{granite}} = 1.5739 \cdot 10^{21} \cdot \text{kg}$$

$$m_U := m_{\text{granite}} \cdot 4 \cdot 10^{-6} \quad m_U = 6.2957 \cdot 10^{15} \cdot \text{kg}$$

$$m_{\text{Th}} := m_{\text{granite}} \cdot 18 \cdot 10^{-6} \quad m_{\text{Th}} = 2.8331 \cdot 10^{16} \cdot \text{kg}$$

$$m_K := m_{\text{granite}} \cdot 3.6 \cdot 10^{-2} \quad m_K = 5.6662 \cdot 10^{19} \cdot \text{kg} \quad f_{K40} := 0.0117 \cdot \%$$

$$m_{U235} := m_U \cdot 0.72 \cdot 10^{-2} \quad m_{U235} = 4.5329 \cdot 10^{13} \cdot \text{kg}$$

$$m_{U238} := m_U \cdot 99.2745 \cdot 10^{-2} \quad m_{U238} = 6.2501 \cdot 10^{15} \cdot \text{kg}$$

$$m_{\text{Th232}} := m_{\text{Th}} \quad m_{\text{Th232}} = 2.8331 \cdot 10^{16} \cdot \text{kg}$$

$$m_{K40} := m_K \cdot f_{K40} \quad m_{K40} = 6.6294 \cdot 10^{15} \cdot \text{kg}$$

$$t_{h235} := 7.038 \cdot 10^8 \cdot \text{yr} \quad M_{235} := 235 \cdot \text{gm} \cdot \text{mole}^{-1} \quad \lambda_{235} := \frac{\ln(2)}{t_{h235}}$$

$$t_{h238} := 4.468 \cdot 10^9 \cdot \text{yr} \quad M_{238} := 238 \cdot \text{gm} \cdot \text{mole}^{-1} \quad \lambda_{238} := \frac{\ln(2)}{t_{h238}}$$

$$t_{h232} := 1.405 \cdot 10^{10} \cdot \text{yr} \quad M_{232} := 232 \cdot \text{gm} \cdot \text{mole}^{-1} \quad \lambda_{232} := \frac{\ln(2)}{t_{h232}}$$

$$t_{h40} := 1.28 \cdot 10^9 \cdot \text{yr} \quad M_{40} := 40 \cdot \text{gm} \cdot \text{mole}^{-1} \quad \lambda_{40} := \frac{\ln(2)}{t_{h40}}$$

$$R_{235} := \lambda_{235} \cdot \frac{m_{U235}}{M_{235}} \cdot N_A \quad R_{238} := \lambda_{238} \cdot \frac{m_{U238}}{M_{238}} \cdot N_A$$

$$R_{232} := \lambda_{232} \cdot \frac{m_{\text{Th232}}}{M_{232}} \cdot N_A \quad R_{40} := \lambda_{40} \cdot \frac{m_{K40}}{M_{40}} \cdot N_A$$

$$Q_{235} := \left( 4.6785 + \frac{1}{3} \cdot 0.3 + 5.1482 + \frac{1}{3} \cdot 0.04 + 6.14649 + 5.9792 + 6.9461 + 7.5264 + \frac{1}{3} \cdot 1.4 + 6.7511 + \frac{1}{3} \cdot 1.4 \right) \cdot 1.6021773 \cdot 10^{-13} \cdot \text{joule}$$

$$Q_{235} = 7.0853 \cdot 10^{-12} \cdot \text{joule}$$

$$P_{235} := R_{235} \cdot Q_{235} \quad P_{235} = 2.5686 \cdot 10^{10} \cdot \text{watt}$$

$$Q_{238} := \left[ 4.274 + 4.8585 + 4.7711 + 4.87067 + 5.5904 + 6.11473 + 7.83353 + 5.4075 \dots \right] \cdot 1.602 \cdot 10^{-13} \text{ joule} \\ + \frac{1}{3} \cdot (0.2 + 2.3 + 0.7 + 1.5 + 0.02 + 1.2)$$

$$Q_{238} = 7.3201 \cdot 10^{-12} \text{ joule}$$

$$P_{238} := R_{238} Q_{238} \quad P_{238} = 5.6909 \cdot 10^{11} \text{ watt}$$

$$Q_{232} := \left[ 4.083 + 5.52014 + 5.78890 + 6.40472 + 6.9064 + 8.95336 + \frac{1}{3} \cdot (0.04 + 1.2 + 0.3 + 2.3) \right] \cdot 1.6021773 \cdot 10^{-13} \text{ joule}$$

$$Q_{232} = 6.2383 \cdot 10^{-12} \text{ joule}$$

$$P_{232} := R_{232} Q_{232} \quad P_{232} = 7.1719 \cdot 10^{11} \text{ watt}$$

$$Q_{40} := \frac{1.3}{3} \cdot 1.6021773 \cdot 10^{-13} \text{ joule}$$

$$P_{40} := R_{40} Q_{40} \quad P_{40} = 1.1891 \cdot 10^{11} \text{ watt}$$

$$\text{Area} := 4 \cdot \pi r_y^2 \quad P_{tot} := P_{235} + P_{238} + P_{232} + P_{40}$$

$$\frac{P_{tot}}{\text{Area}} = 0.0028 \cdot \text{m}^{-2} \text{ watt}$$

$$\text{Area} = 5.1006 \cdot 10^{14} \text{ m}^2$$

$$\frac{P_{235}}{\text{Area}} = 5.0357 \cdot 10^{-5} \text{ m}^{-2} \text{ watt}$$

$$\frac{P_{238}}{\text{Area}} = 0.0011 \cdot \text{m}^{-2} \text{ watt}$$

$$\frac{P_{232}}{\text{Area}} = 0.0014 \cdot \text{m}^{-2} \text{ watt}$$

$$\frac{P_{40}}{\text{Area}} = 2.3312 \cdot 10^{-4} \text{ m}^{-2} \text{ watt}$$

$$\frac{P_{235} + P_{238}}{\text{Area}} = 0.0012 \cdot \frac{\text{watt}}{\text{m}^2}$$