(17.3) ^{199}Au can be formed through two successive n,γ-reactions on ^{197}Au (100% in nature). If 1 g ^{197}Au is irradiated with 10^{18} n m^{-2} s^{-1} during 30 h, what will the disintegration rate of ^{199}Au be at the end of the irradiation? The chain or events to be considered is:

^{197}Au ($\sigma_{n,\gamma}$ 98.8 b) --> ^{198}Au ($t_{½}$ 2.694 d, $\sigma_{n,\gamma}$ 25100 b) --> ^{199}Au ($t_{½}$ 3.139 d, $\sigma_{n,\gamma}$ 30 b). -->
Neglect self shielding.

First some definitions:

$Bq := sec^{-1}$ $\quad N_A := 6.022137 \cdot 10^{23} \cdot mole^{-1}$ $\quad barn := 10^{-28} \cdot m^2$

Then the given values from the text:

$m_{197} := 1 \cdot gm$ $\quad \phi := 10^{18} \cdot m^{-2} \cdot sec^{-1}$ $\quad t_{irr} := 30 \cdot hr$

Then other data from the book and chart of nuclides:

$t_{h198} := 2.694 \cdot day$ $\quad t_{h199} := 3.139 \cdot day$ $\quad \lambda_{198} := \dfrac{ln(2)}{t_{h198}}$ $\quad \lambda_{199} := \dfrac{ln(2)}{t_{h199}}$

$\sigma_{197} := 98.8 \cdot barn$ $\quad \sigma_{198} := 25100 \cdot barn$ $\quad \sigma_{199} := 30 \cdot barn$

$M_{197} := 196.97 \cdot gm \cdot mole^{-1}$

Calculate the total number of gold atoms present:

$$N_{197} := \frac{m_{197}}{M_{197}} \cdot N_A$$

Solution of the series of differential equations with all initial amounts (except ^{197}Au) as zero at t=0 gives:

$k_a := \phi \cdot \sigma_{197} \cdot N_{197}$ $\quad k_b := \lambda_{198} + \phi \cdot \sigma_{198}$ $\quad k_d := \lambda_{199} + \phi \cdot \sigma_{199}$

$$A_{199} := \lambda_{199} \cdot \left[\frac{\phi \cdot \sigma_{198} \cdot k_a}{k_b \cdot k_d} \cdot \left(1 - exp\left(-k_d \cdot t_{irr}\right)\right) + \frac{\phi \cdot \sigma_{198} \cdot k_a}{k_b \cdot \left(k_b - k_d\right)} \cdot \left(exp\left(-k_b \cdot t_{irr}\right) - exp\left(-k_d \cdot t_{irr}\right)\right) \right]$$

$$A_{199} = 8.521 \cdot 10^{11} \cdot Bq$$