(17.4) ^{246}Cm has a half-life of 4730 y. It can be obtained through neutron capture in ^{245}Cm, which has a half-life of 8500 y; the reaction cross-section is 345 b. Both isotopes are also fissioned by thermal neutrons, σ_{245} 2020 b and σ_{246} 0.17 b. ^{246}Cm also has a n,γ cross-section of 1.3 b. Because one does not want to loose too much ^{245}Cm, the irradiation is timed to give a maximum yield of ^{246}Cm. If the neutron flux is $2*10^{18}$ n m^{-2} s^{-1}, (a) when does the ^{246}Cm concentration reach its maximum? (b) What is the ratio between ^{246}Cm produced and ^{245}Cm consumed at that time?

The reaction chain is:

^{245}Cm ($t_{1/2}$ 8500 y, $\sigma_{n,\gamma}$ 345 b, $\sigma_{n,f}$ 2020 b) --> ^{246}Cm ($t_{1/2}$ 4730 y, $\sigma_{n,\gamma}$ 1.3 b, $\sigma_{n,f}$ 0.17 b)

First definitions and data given in the text:

$barn := 10^{-28} \cdot m^2$ $\quad \phi := 2 \cdot 10^{18} \cdot m^{-2} \cdot sec^{-1}$

$t_{h245} := 8500 \cdot yr$ $\quad \lambda_{245} := \frac{ln(2)}{t_{h245}}$ $\quad \sigma_{\gamma 245} := 345 \cdot barn$ $\quad \sigma_{f245} := 2020 \cdot barn$

$t_{h246} := 4730 \cdot yr$ $\quad \lambda_{246} := \frac{ln(2)}{t_{h246}}$ $\quad \sigma_{\gamma 246} := 1.3 \cdot barn$ $\quad \sigma_{f246} := 0.17 \cdot barn$

Then the calculations: $N_{0245} := 10^{24}$ An arbitrary number of initial atoms is assumed, e.g. 10^{24}.

$k_a := \lambda_{245} + \phi \cdot (\sigma_{\gamma 245} + \sigma_{f245})$ $\quad k_b := \lambda_{246} + \phi \cdot (\sigma_{\gamma 246} + \sigma_{f246})$

(a) From the derivative of the amount of ^{246}Cm vs time:

$$t_{max} := \frac{ln\left(\frac{k_a}{k_b}\right)}{k_a - k_b} \qquad t_{max} = 1.559 \cdot 10^7 \cdot sec \quad \text{or} \quad t_{max} = 180.395 \cdot day$$

(b) From the solutions to the differential equations and t_{max}:

$N_{245} := N_{0245} \cdot exp(-k_a \cdot t_{max})$

$$N_{246} := \frac{\phi \cdot \sigma_{\gamma 245} \cdot N_{0245}}{k_a - k_b} \cdot \left(exp(-k_b \cdot t_{max}) - exp(-k_a \cdot t_{max})\right)$$

$\Delta_{245} := N_{0245} - N_{245}$ $\quad Ratio := \frac{N_{246}}{\Delta_{245}}$ $\quad Ratio = 0.145$