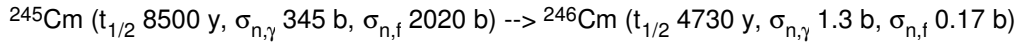


(17.4) ^{246}Cm has a half-life of 4730 y. It can be obtained through neutron capture in ^{245}Cm , which has a half-life of 8500 y; the reaction cross-section is 345 b. Both isotopes are also fissioned by thermal neutrons, σ_{245} 2020 b and σ_{246} 0.17 b. ^{246}Cm also has a n,γ cross-section of 1.3 b. Because one does not want to lose too much ^{245}Cm , the irradiation is timed to give a maximum yield of ^{246}Cm . If the neutron flux is $2 \cdot 10^{18} \text{ n m}^{-2} \text{ s}^{-1}$, (a) when does the ^{246}Cm concentration reach its maximum? (b) What is the ratio between ^{246}Cm produced and ^{245}Cm consumed at that time?

The reaction chain is:



First definitions and data given in the text:

$$\text{barn} := 10^{-28} \cdot \text{m}^2 \quad \phi := 2 \cdot 10^{18} \cdot \text{m}^{-2} \cdot \text{sec}^{-1}$$

$$t_{h245} := 8500 \cdot \text{yr} \quad \lambda_{245} := \frac{\ln(2)}{t_{h245}} \quad \sigma_{\gamma 245} := 345 \cdot \text{barn} \quad \sigma_{f245} := 2020 \cdot \text{barn}$$

$$t_{h246} := 4730 \cdot \text{yr} \quad \lambda_{246} := \frac{\ln(2)}{t_{h246}} \quad \sigma_{\gamma 246} := 1.3 \cdot \text{barn} \quad \sigma_{f246} := 0.17 \cdot \text{barn}$$

Then the calculations: $N_{0245} := 10^{24}$ An arbitrary number of initial atoms is assumed, e.g. 10^{24} .

$$k_a := \lambda_{245} + \phi \cdot (\sigma_{\gamma 245} + \sigma_{f245}) \quad k_b := \lambda_{246} + \phi \cdot (\sigma_{\gamma 246} + \sigma_{f246})$$

(a) From the derivative of the amount of ^{246}Cm vs time:

$$t_{max} := \frac{\ln\left(\frac{k_a}{k_b}\right)}{k_a - k_b} \quad t_{max} = 1.559 \cdot 10^7 \cdot \text{sec} \quad \text{or} \quad t_{max} = 180.395 \cdot \text{day}$$

(b) From the solutions to the differential equations and t_{max} :

$$N_{245} := N_{0245} \exp(-k_a \cdot t_{max})$$

$$N_{246} := \frac{\phi \cdot \sigma_{\gamma 245} \cdot N_{0245}}{k_a - k_b} \cdot (\exp(-k_b \cdot t_{max}) - \exp(-k_a \cdot t_{max}))$$

$$\Delta_{245} := N_{0245} - N_{245} \quad \text{Ratio} := \frac{N_{246}}{\Delta_{245}} \quad \text{Ratio} = 0.145$$