(19.15) Consider a power reactor in which micro-spheres (r = 0.3 mm) of frozen 1:1 T-D mixture (density 170 kg m^{-3}) are fused by laser irradiation. The laser compresses the spheres to $N_v = N_v^o*10^4$, where N_v^o is the number of atoms per m^3 at ordinary pressure, and also heats it to a temperature corresponding to almost 20 keV. The energy developed through the T-D fusion reaction leads to expansion of the spheres, which occurs with the velocity of sound ($v_s = 10^8$ m s^{-1}). This leads to no more than 25% of the particles fusing. (a) By what factor will the Lawson criterion be exceeded? (b) What power is produced if the fusion micro-explosions occur at a rate of 30 per s?

$N_A := 6.022137\cdot 10^{23}\cdot mole^{-1}$ $\quad k := 1.38066\cdot 10^{-23}\cdot joule\cdot K^{-1}$ $\quad keV := 1.6021773\cdot 10^{-19+3}\cdot joule$

$R_{gas} := 8.31451\cdot joule\cdot mole^{-1}\cdot K^{-1}$

$\rho_0 := 170\cdot kg\cdot m^{-3}$ $\quad M_{TD} := 5\cdot gm\cdot mole^{-1}$ $\quad r_0 := 0.3\cdot mm$

$N_{v0} := \frac{\rho_0\cdot N_A\cdot 2}{M_{TD}}$ $\quad N_{v0} = 4.095\cdot 10^{28}\ \cdot m^{-3}$ $\quad N_v := N_{v0}\cdot 10^4$

$V_0 := \frac{4\cdot\pi\cdot r_0^3}{3}$ $\quad V_0 = 1.131\cdot 10^{-10}\ \cdot m^3$ $\quad v_s := 10^8\cdot m\cdot sec^{-1}$

$V_{init} := \frac{N_{v0}}{N_v}\cdot V_0$ $\quad V_{init} = 1.131\cdot 10^{-14}\ \cdot m^3$

$r_{init} := \left(\frac{3}{4}\cdot\frac{V_{init}}{\pi}\right)^{\frac{1}{3}}$ $\quad r_{init} = 1.392\cdot 10^{-5}\ \cdot m$

$N_{bead} := N_{v0}\cdot V_0$ $\quad N_{bead} = 4.631\cdot 10^{18}$

$E_0 := 20\cdot keV$ $\quad T_0 := \frac{2}{3}\cdot\frac{E_0}{k}$ $\quad T_0 = 1.547\cdot 10^8\ \cdot K$

$E_{cbmin} := \frac{1.109\cdot(2+3)\cdot 1\cdot 1}{3\cdot\left(3^{\frac{1}{3}}+2^{\frac{1}{3}}\right)}\cdot 10^3\cdot keV$ $\quad E_{cbmin} = 684.018\ \cdot keV$ $\quad$ Hence, only reaction by tunneling is possible.

At a particle density of 10^{15} cm^{-3}, ignition temperature is 45 MK for a D-T system.

$T_{ign} := 45\cdot 10^6\cdot K$ $\quad E_{ign} := \frac{3}{2}\cdot T_{ign}\cdot k$ $\quad E_{ign} = 5.817\ \cdot keV$

Hence, we will be above the ignition temperature before expansion starts.

The specific energy production rate (W/m^3) from 10^7 to about 10^8 K will be given by:

$k_r := 5.5171\cdot 10^{-31}\cdot kg\cdot m^5\cdot sec^{-3}$

$\frac{1}{}$

$k_t := 4.2\cdot 10^3\cdot K^3$

$P_V(N,T) := k_r\cdot N^2\cdot \exp\left(-k_t\cdot T^{-\frac{1}{3}}\right)$

where N is number of particles/m3 of each kind and *T* is temperature in K.

cont.

$U(N,T) := 3\cdot N\cdot R_{gas}\cdot T$

$r(t) := r_{init} + v_s\cdot t$ $V(t) := \frac{4}{3}\cdot\pi\cdot r(t)^3$ $N(t) := \frac{N_v}{2}\cdot\frac{1}{V(t)}$

(a) The confinement time can be estimated from the expension time until the reactions stops.

(b)

$Q_{fusion} := 17.58\cdot 10^3\cdot keV$ $yield := 25\cdot\%$ $rate := 30\cdot sec^{-1}$

$P_{react} := \frac{N_{bead}}{2}\cdot yield\cdot Q_{fusion}\cdot rate$ $P_{react} = 4.892\cdot 10^7\ \cdot watt$