CHAPTER 3

# *Nuclear Mass and Stability*

**Contents**



## 3.1. Patterns of nuclear stability

There are approximately 275 different nuclei which have shown no evidence of radioactive decay and, hence, are said to be stable with respect to radioactive decay. When these nuclei are compared for their constituent nucleons, we find that approximately 60% of them have both an even number of protons and an even number of neutrons (*even-even nuclei*). The remaining 40% are about equally divided between those that have an even number of protons and an odd number of neutrons (*even-odd nuclei*) and those with an odd number of protons and an even number of neutrons (*odd-even nuclei*). There are only 5 stable nuclei known which have both an odd number of protons and odd number of neutrons (*odd-odd nuclei*); $^{2}_{1}$H, $^{6}_{3}$Li, $^{10}_{5}$B, $^{14}_{7}$N, and $^{50}_{23}$V. It is significant that the first stable odd-odd nuclei are abundant in the very light elements (the low abundance of $^{2}_{1}$H has a special explanation, see Ch. 17). The last nuclide is found in low isotopic abundance (0.25%) and we cannot be certain that this nuclide is not unstable to radioactive decay with extremely long half-life.

Considering this pattern for the stable nuclei, we can conclude that nuclear stability is favored by even numbers of protons and neutrons. The validity of this statement can be confirmed further by considering for any particular element the number and types of stable isotopes; see Figure 3.1. Elements of even atomic number (i.e. even number of protons) are characterized by having a relatively sizable number of stable isotopes, usually 3 or more. For example, the element tin, atomic number 50, has 10 stable isotopes while

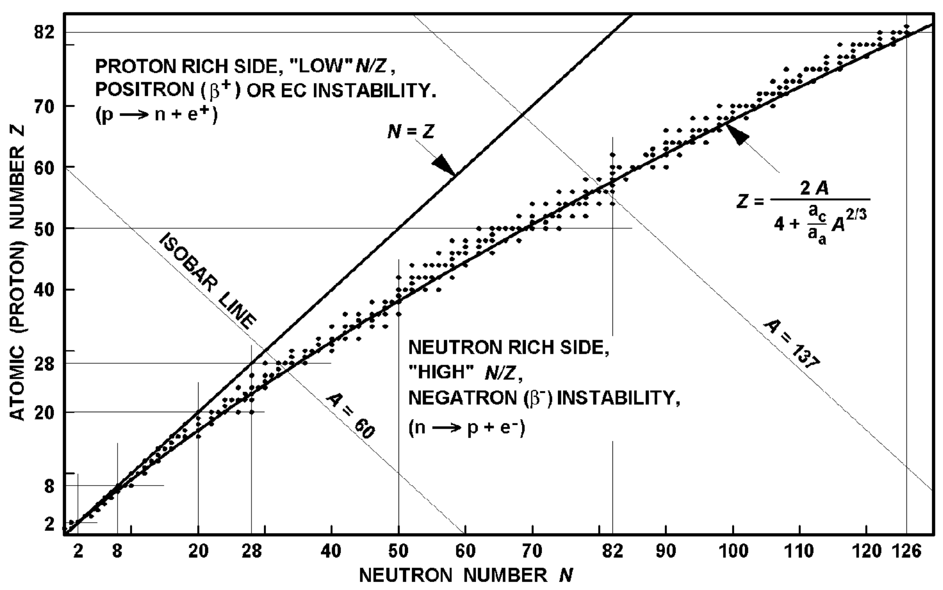


FIG. 3.1. Chart of stable nuclides as a function of their proton (*Z*) and neutron (*N*) numbers. The numbers denoted 2, 8, etc., are discussed in Chapter 11.

cadmium ($Z = 48$) and telluriu($Z = 52$) each have 8. By contrast silver ($Z = 47$) and antimony ($Z = 51$) each have only 2 stable isotopes, and rhodium ($Z = 45$), indium ($Z = 49$), and iodine ($Z = 53$) have only 1 stable isotope. Many other examples of the extra stabilization of even numbers of nucleons can be found from a detailed examination of Figure 3.1, or, easier, from nuclide charts, e.g. Appendix C. The guide lines of *N* and *Z* equal to 2, 8, 20, etc., have not been selected arbitrarily. These proton and neutron numbers represent unusually stable proton and neutron configurations, as will be discussed further in Chapter 11. The curved line through the experimental points is calculated based on the liquid drop model of the nucleus which is discussed later in this chapter.

Elements of odd *Z* have none, one or two stable isotopes, and their stable isotopes have an even number of neutrons, except for the 5 odd-odd nuclei mentioned above. This is in contrast to the range of stable isotopes of even *Z*, which includes nuclei of both even and odd *N*, although the former outnumber the latter. Tin ($Z = 50$), for example, has 7 stable even-even isotopes and only 3 even-odd ones.

The greater number of stable nuclei with even numbers of protons and neutrons is explained in terms of the energy stabilization gained by combination of like nucleons to form pairs, i.e. protons with protons and neutrons with neutrons, but not protons with neutrons. If a nucleus has, for example, an even number of protons, all these protons can exist in pairs. However, if the nucleus has an odd number of protons, at least one of these protons must exist in an unpaired state. The increase in stability resulting from complete pairing in elements of even *Z* is responsible for their ability to accommodate a greater range of neutron numbers as illustrated for the isotopes of germanium ($_{32}$Ge, 5 stable isotopes), relative to those of gallium ($_{31}$Ga, 2 stable isotopes), and arsenic ($_{33}$As, 1 stable isotope). The same pairing stabilization holds true for neutrons so that an even-even nuclide which has all its nucleons, both neutrons and protons, paired represents a quite stable situation. In the elements in which the atomic number is even, if the neutron number is uneven, there is still some stability conferred through the proton-proton pairing. For elements of odd atomic number, unless there is stability due to an even neutron number (neutron-neutron pairing), the nuclei are radioactive with rare exceptions. We should also note that the number of stable nuclear species is approximately the same for even-odd and odd-even cases. The pairing of protons with protons and neutrons with neutrons must thus confer approximately equal degrees of stability to the nucleus.

## 3.2. Neutron to proton ratio

If a graph is made (Fig. 3.1)[1] of the relation of the number of neutrons to the number of protons in the known stable nuclei, we find that in the light elements stability is achieved when the number of neutrons and protons are approximately equal ($N = Z$). However, with increasing atomic number of the element (i.e. along the $Z$-line), the ratio of neutrons to protons, the $N/Z$ ratio, for nuclear stability increases from unity to approximately 1.5 at bismuth. Thus pairing of the nucleons is not a sufficient criterion for stability: a certain ratio $N/Z$ must also exist. However, even this does not suffice for stability, because at high $Z$-values, a new mode of radioactive decay, α-emission, appears. Above bismuth the nuclides are all unstable to radioactive decay by α-particle emission, while some are unstable also to β-decay.

If a nucleus has a $N/Z$ ratio too high for stability, it is said to be *neutron-rich*. It will undergo radioactive decay in such a manner that the neutron to proton ratio decreases to approach more closely the stable value. In such a case the nucleus must decrease the value of $N$ and increase the value of $Z$, which can be done by conversion of a neutron to a proton. When such a conversion occurs within a nucleus, $\beta^-$ (or *negatron*) *emission* is the consequence, with creation and emission of a negative β-particle designated by $\beta^-$ or $_{-1}^{0}e$ (together with an anti-neutrino, here omitted for simplicity, see Ch. 4). For example:

$$^{116}_{49}\text{In} \rightarrow {}^{116}_{50}\text{Sn} + {}^{0}_{-1}\text{e}^-$$

At extreme $N/Z$ ratios beyond the so called neutron drip-line, or for highly excited nuclei, *neutron emission* is an alternative to $\beta^-$ decay.

If the $N/Z$ ratio is too low for stability, then radioactive decay occurs in such a manner as to lower $Z$ and increase $N$ by conversion of a proton to neutron. This may be accomplished through *positron emission*, i.e. creation and emission of a positron ($\beta^+$ or $_{+1}^{0}e$), or by absorption by the nucleus of an orbital electron (*electron capture*, EC). Examples of these reactions are:

$$^{116}_{51}\text{Sb} \rightarrow {}^{116}_{50}\text{Sn} + {}^{0}_{+1}\text{e}^+ \quad \text{and} \quad {}^{195}_{79}\text{Au} + {}^{0}_{-1}\text{e}^- \xrightarrow{\text{EC}} {}^{195}_{78}\text{Pt}$$

[1] In graphs like Fig. 3.1, $Z$ is commonly plotted as the abscissa; we have here reversed the axes to conform with the commercially available isotope and nuclide charts.

Positron emission and electron capture are competing processes with the probability of the latter increasing as the atomic number increases. Beta decay is properly used to designate all three processes, $\beta^-$, $\beta^+$, and EC. (The term "beta decay" without any specification usually only refers to $\beta^-$ emission.)

Thus in the early part of the Periodic Table, unstable neutron deficient nuclides decay by positron emission, but for the elements in the platinum region and beyond, decay occurs predominantly by electron capture. Both processes are seen in isotopes of the elements in the middle portion of the Periodic Table, see Figure 3.1 and Appendix C.

An alternative to positron decay (or EC) is *proton emission*, which, although rare, has been observed in about 40 nuclei very far off the stability line. These nuclei all have half-lives $\leq$ 1 min. For example: $^{115}$Xe, $t_{1/2}$ (p) 18 s; proton/EC ratio, $3 \times 10^{-3}$.

We can understand why the $N/Z$ ratio must increase with atomic number in order to have nuclear stability when we consider that the protons in the nucleus must experience a repulsive Coulomb force. The fact that stable nuclei exist means that there must be an attractive force tending to hold the neutrons and protons together. This attractive nuclear force must be sufficient in stable nuclei to overcome the disruptive Coulomb force. Conversely, in unstable nuclei there is a net imbalance between the attractive nuclear force and the disruptive Coulomb force. As the number of protons increases, the total repulsive Coulomb force must increase. Therefore, to provide sufficient attractive force for stability the number of neutrons increases more rapidly than that of the protons.

Neutrons and protons in nuclei are assumed to exist in separate *nucleon orbitals* just as electrons are in electron orbitals in atoms. If the number of neutrons is much larger than the number of protons, the neutron orbitals occupied extend to higher energies than the highest occupied proton orbital. As $N/Z$ increases, a considerable energy difference can develop between the last (highest energy) neutron orbital filled and the last proton orbital filled. The stability of the nucleus can be enhanced when an odd neutron in the highest

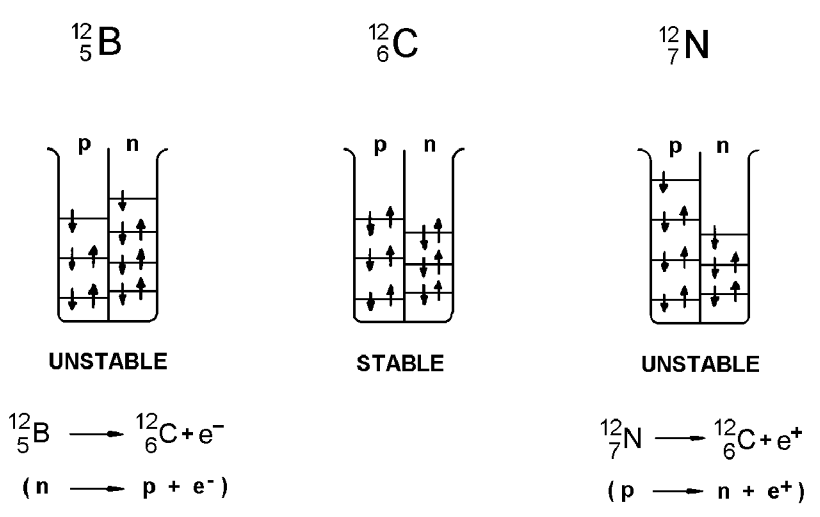


FIG. 3.2. The separation and pairing of nucleons in assumed energy levels within the isobar $A = 12$. Half-life for the unstable $^{12}$B is 0.02 s, and for $^{12}$N 0.01 s.

neutron orbital is transformed into a proton fitting into a vacant lower energy proton orbital; see the example for $A = 12$ in Figure 3.2. These questions of nuclear forces and the energy levels of nucleons are discussed more extensively in Chapter 11.

### 3.3. Mass defect

It was noted in Chapter 1 that the masses of nuclei (in u) are close to the mass number $A$. Using the mass of carbon-12 as the basis ($^{12}_{6}C \equiv 12$ u), the hydrogen atom and the neutron do not have exactly unit masses. We would expect that the mass $M_A$ of an atom with mass number $A$ would be given by the number of protons ($Z$) times the mass of the hydrogen atom ($\mathbf{M}_H$) plus the number of neutrons ($N$) times the mass of the neutron ($\mathbf{M}_n$), i.e.

$$M_A \approx Z\mathbf{M}_H + N\mathbf{M}_n \tag{3.1}$$

For deuterium with one neutron and one proton in the nucleus, we would then anticipate an atomic mass of

$$\mathbf{M}_H + \mathbf{M}_n = 1.007\,825 + 1.008\,665 = 2.016\,490 \text{ u}$$

When the mass of the deuterium atom is measured, it is found to be 2.014 102 u. The difference between the measured and calculated mass values, which in the case of deuterium equals −0.002 388 u, is called the *mass defect* ($\Delta M_A$):

$$\Delta M_A = M_A - Z\mathbf{M}_H - N\mathbf{M}_n \tag{3.2}$$

From the Einstein equation, $E = m\mathbf{c}^2$, which is discussed further in Chapters 4 and 12, one can calculate that one atomic mass unit is equivalent to 931.5 MeV, where MeV is a million electron volts.

$$E = m\mathbf{c}^2 = 931.5\,\Delta M_A \tag{3.3}$$

The relationship of energy and mass would indicate that in the formation of deuterium by the combination of a proton and neutron, the mass defect of 0.002 388 u would be observed as the liberation of an equivalent amount of energy, i.e. $931.5 \times 0.002\,388 = 2.224$ MeV. Indeed, the emission of this amount of energy (in the form of γ-rays) is observed when a proton captures a low energy neutron to form $^{2}_{1}H$. As a matter of fact, in this particular case, the energy liberated in the formation of deuterium has been used in the reverse calculation to obtain the mass of the neutron since it is not possible to determine directly the mass of the free neutron With the definition (3.2) all stable nuclei are found to have negative $\Delta M_A$ values; thus the term "defect".

In nuclide (or isotope) tables the neutral atomic mass is not always given, but instead the *mass excess* (often, unfortunately, also called mass defect). We indicate this as $\delta_A$ and define it as the difference between the measured mass and the mass number of the particular atom:

TABLE 3.1. *Atomic masses and binding energies*.

| Element | *Z* | *N* | *A* | Atomic mass $M_A$ (u) | Mass excess $M_A$ - *A* (μu) | Mass defect $\Delta M_A$ (μu) | Binding energy $E_B$ (MeV) | $E_B/A$ (MeV/*A*) |
|---|---|---|---|---|---|---|---|---|
| n | 0 | 1 | 1 | 1.008 665 | 8 665 | 0 | - | - |
| H | 1 | 0 | 1 | 1.007 825 | 7 825 | 0 | - | - |
| D | 1 | 1 | 2 | 2.014 102 | 14 102 | -2 388 | 2.22 | 1.11 |
| T | 1 | 2 | 3 | 3.016 049 | 16 049 | -9 106 | 8.48 | 2.83 |
| He | 2 | 1 | 3 | 3.016 029 | 16 029 | -8 286 | 7.72 | 2.57 |
| He | 2 | 2 | 4 | 4.002 603 | 2 603 | -30 377 | 28.30 | 7.07 |
| He | 2 | 4 | 6 | 6.018 886 | 18 886 | -31 424 | 29.27 | 4.88 |
| Li | 3 | 3 | 6 | 6.015 121 | 15 121 | -34 348 | 32.00 | 5.33 |
| Li | 3 | 4 | 7 | 7.016 003 | 16 003 | -42 132 | 39.25 | 5.61 |
| Be | 4 | 3 | 7 | 7.016 928 | 16 928 | -40 367 | 37.60 | 5.37 |
| Be | 4 | 5 | 9 | 9.012 182 | 12 182 | -62 442 | 58.16 | 6.46 |
| Be | 4 | 6 | 10 | 10.013 534 | 13 534 | -69 755 | 64.98 | 6.50 |
| B | 5 | 5 | 10 | 10.012 937 | 12 937 | -69 513 | 64.75 | 6.48 |
| B | 5 | 6 | 11 | 11.009 305 | 9 305 | -81 809 | 76.20 | 6.93 |
| C | 6 | 6 | 12 | 12.000 000 | 0 | -98 940 | 92.16 | 7.68 |
| N | 7 | 7 | 14 | 14.003 074 | 3 074 | -112 356 | 104.7 | 7.48 |
| O | 8 | 8 | 16 | 15.994 915 | -5 085 | -137 005 | 127.6 | 7.98 |
| F | 9 | 10 | 19 | 18.998 403 | -1 597 | -158 671 | 147.8 | 7.78 |
| Ne | 10 | 10 | 20 | 19.992 436 | -7 564 | -172 464 | 160.6 | 8.03 |
| Na | 11 | 12 | 23 | 22.989 768 | -10 232 | -200 287 | 186.6 | 8.11 |
| Mg | 12 | 12 | 24 | 23.985 042 | -14 958 | -212 837 | 198.3 | 8.26 |
| Al | 13 | 14 | 27 | 26.981 539 | -18 461 | -241 495 | 225.0 | 8.33 |
| Si | 14 | 14 | 28 | 27.976 927 | -23 073 | -253 932 | 236.5 | 8.45 |
| P | 15 | 16 | 31 | 30.973 762 | -26 238 | -282 252 | 262.9 | 8.48 |
| K | 19 | 20 | 39 | 38.963 707 | -36 293 | -358 266 | 333.7 | 8.56 |
| Co | 27 | 32 | 59 | 58.933 198 | -66 802 | -555 355 | 517.3 | 8.77 |
| Zr | 40 | 54 | 94 | 93.906 315 | -93 685 | -874 591 | 814.7 | 8.67 |
| Ce | 58 | 82 | 140 | 139.905 433 | -94 567 | -1 258 941 | 1 172.7 | 8.38 |
| Ta | 73 | 108 | 181 | 180.947 993 | -52 007 | -1 559 045 | 1 452.2 | 8.02 |
| Hg | 80 | 119 | 199 | 198.968 254 | -31 746 | -1 688 872 | 1 573.2 | 7.91 |
| Th | 90 | 142 | 232 | 232.038 051 | 38 051 | -1 896 619 | 1 766.7 | 7.62 |
| U | 92 | 143 | 235 | 235.043 924 | 43 924 | -1 915 060 | 1 783.9 | 7.59 |
| U | 92 | 144 | 236 | 236.045 563 | 45 563 | -1 922 087 | 1 790.4 | 7.59 |
| U | 92 | 146 | 238 | 238.050 785 | 50 785 | -1 934 195 | 1 801.7 | 7.57 |
| Pu | 94 | 146 | 240 | 240.053 808 | 53 808 | -1 946 821 | 1 813.5 | 7.56 |

$$\delta_A = M_A - A \tag{3.4}$$

Mass excess values are either given in u (or, more commonly, in micro mass units, μu) or in eV (usually keV). Table 3.1 contains a number of atomic masses, mass excess, and mass defect values, as well as some other information which is discussed in later sections.

When two elements form a compound in a chemical system, the amount of heat liberated is a measure of the stability of the compound. The greater this heat of formation (enthalpy, $\Delta H$) the greater the stability of the compound. When carbon is combined with oxygen to form $CO_2$, it is found experimentally that 393 kJ of heat is evolved per mole of $CO_2$ formed. If we use the Einstein relationship, we can calculate that this would correspond to a total mass loss of $4.4 \times 10^{-9}$ g for each mole of $CO_2$ formed (44 g). Although chemists do not doubt that this mass loss actually occurs, at present there are no instruments of sufficient sensitivity to measure such small changes.

The energy changes in nuclear reactions are much larger. This can be seen if we use the relationship between electron volts and joules (or calories) in Appendix IV, and observe that nuclear reaction formulas and energies refer to single atoms (or molecules), while chemical reactions and equations refer to number of moles; we have:

$$1 \text{ eV/molecule} = 1.6022 \times 10^{-19} \times 6.0221 \times 10^{23} = 96.48 \text{ kJ mole}^{-1} = 3.8268 \times 10^{-20} \times 6.0221 \times 10^{23} = 23.045 \text{ kcal mole}^{-1}$$

Thus, the formation of deuterium from a neutron and a hydrogen atom would lead to the liberation of $214.6 \times 10^6$ kJ ($51.3 \times 10^6$ kcal) for each mole of deuterium atoms formed. By comparison, then, the nuclear reaction leading to the formation of deuterium is approximately half a million times more energetic than the chemical reaction leading to formation of $CO_2$.

It is not common practice to use mole quantities in considering nuclear reactions as the number of individual reactions under laboratory conditions is well below $6.02 \times 10^{23}$. Therefore, in nuclear science one uses the energy and mass changes involved in the reaction of individual particles and nuclei.

### 3.4. Binding energy

The energy liberated in the formation of $CO_2$ from the elements, the heat of formation, is a measure of the stability of the $CO_2$ molecule. The larger the heat of formation the more stable the molecule since the more energy is required to decompose the molecule into its component atoms. Similarly, the energy liberated in the formation of a nucleus from its component nucleons is a measure of the stability of that nucleus. This energy is known as the *binding energy* ($E_B$) and has the same significance in nuclear science as the heat of formation has in chemical thermodynamics. We have seen that the binding energy of deuterium is 2.22 MeV. The $^4_2$He nucleus is composed of 2 neutrons and 2 protons. The measured mass of the $^4$He atom is 4.002 603 u. The mass defect is:

$$\Delta M_{He} = M_{He} - 2\mathbf{M}_H - 2\mathbf{M}_n = 4.002603 - 2 \times 1.007825 - 2 \times 1.008665 = -0.030377 \text{ u}$$

The binding energy between the nucleons in a nucleus follows the simple relation

$$E_B \text{ (MeV)} = -931.5\,\Delta M_A \text{ (u)} \tag{3.5}$$

which is just another form of eqn. (3.3). Thus the binding energy for $^4$He is 28.3 MeV. It is quite unlikely that 2 neutrons and 2 protons would ever collide simultaneously to form a $^4$He nucleus; nevertheless, this calculation is useful because it indicates that to break $^4$He into its basic component nucleons would require at least 28.3 MeV.

A better indication of the relative stability of nuclei is obtained when the binding energy is divided by the total number of nucleons to give the binding energy per nucleon, $E_B/A$. For $^4$He the value of $E_B/A$ is 28.3/4 or 7.1 MeV, whereas for $^2$H it is 1.11 for the bond between the two nucleons. Clearly, the $^4$He nucleus is considerably more stable than the $^2$H nucleus. For most nuclei the values of $E_B/A$ vary in the rather narrow range 5 – 8 MeV. To a first approximation, therefore, $E_B/A$ is relatively constant which means that the total nuclear binding energy is roughly proportional to the total number of nucleons in the nucleus.

Figure 3.3 shows that the $E_B/A$ values increase with increasing mass number up to a maximum around mass number 60 and then decrease. Therefore the nuclei with mass numbers in the region of 60, i.e. nickel, iron, etc., are the most stable. Also in this Figure we see that certain numbers of neutrons and protons form especially stable configurations − this effect is observed as small humps on the curve.

If two nuclides can be caused to react so as to form a new nucleus whose $E_B/A$ value is larger than that of the reacting species, obviously a certain amount of binding energy would be released. The process which is called *fusion* is "exothermic" only for the nuclides of mass number below 60. As an example, we can choose the reaction

$$^{20}_{10}\text{Ne} + ^{20}_{10}\text{Ne} \rightarrow ^{40}_{20}\text{Ca}$$

From Figure 3.3 we estimate that $E_B/A$ for neon is about 8.0 MeV and for calcium about 8.6 MeV. Therefore, in the 2 neon nuclei $2 \times 20 \times 8.0 = 320$ MeV are involved in the binding energy, while $40 \times 8.6 = 344$ MeV binding energy are involved in the calcium nucleus. When 2 neon nuclei react to form the calcium nucleus the difference in the total binding energy of reactants and products is released; the estimate gives $344 - 320 = 24$ MeV; a calculation using measured masses gives 20.75 MeV.

Figure 3.3 also shows that a similar release of binding energy can be obtained if the elements with mass numbers greater than 60 are split into lighter nuclides with higher $E_B/A$ values. Such a process, whereby a nucleus is split into two smaller nuclides, is known as *fission*. An example of such a fission process is the reaction

$$^{236}_{92}\text{U} \rightarrow ^{140}_{54}\text{Xe} + ^{93}_{38}\text{Sr} + 3\text{n}$$

The binding energy per nucleon for the uranium nucleus is 7.6 MeV, while those for the $^{140}$Xe and $^{93}$Sr are 8.4 and 8.7 MeV respectively. The amount of energy released in this fission reaction is approximately $140 \times 8.4 + 93 \times 8.7 - 236 \times 7.6 = 191.5$ MeV for each uranium fission.

### 3.5. Nuclear radius

Rutherford showed by his scattering experiments that the nucleus occupies a very small portion of the total volume of the atom. Roughly, the radii of nuclei vary from 1/10 000 to 1/100 000 of the radii of atoms. While atomic sizes are of the order of 100 pm ($10^{-10}$ m), the common unit of nuclear size is the femtometer (1 fm $= 10^{-15}$ m), sometimes referred to as 1 Fermi.

Experiments designed to study the size of nuclei indicate that the volumes of nuclei ($V_n$) are directly proportional to the total number of nucleons present, i.e.

$$V_n \propto A \tag{3.6}$$

Since for a sphere $V \propto r^3$, where $r$ is the radius of the sphere, for a spherical nucleus $r^3 \propto A$, or $r \propto A^{1/3}$. Using $r_o$ as the proportionality constant

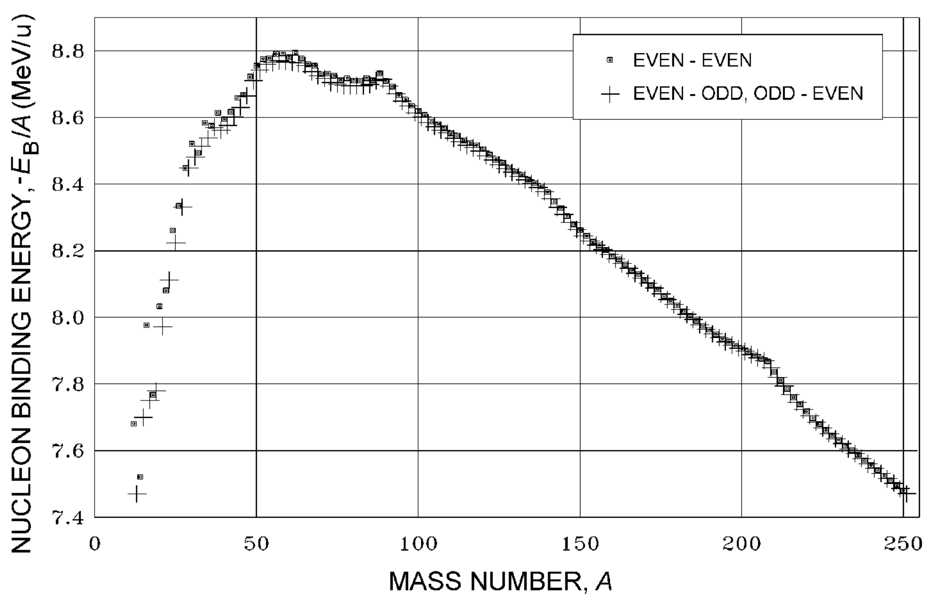


FIG. 3.3. Binding energy per nucleon ($E_B/A$) for the most stable isobars as function of mass number ($A$).

$$r = r_o A^{1/3} \tag{3.7}$$

The implications of this is that the nucleus is composed of nucleons packed closely together with a constant density (about 0.2 nucleons fm$^{-3}$) from the center to the edge of the nucleus. This constant density model of the nucleus has been shown to be not completely

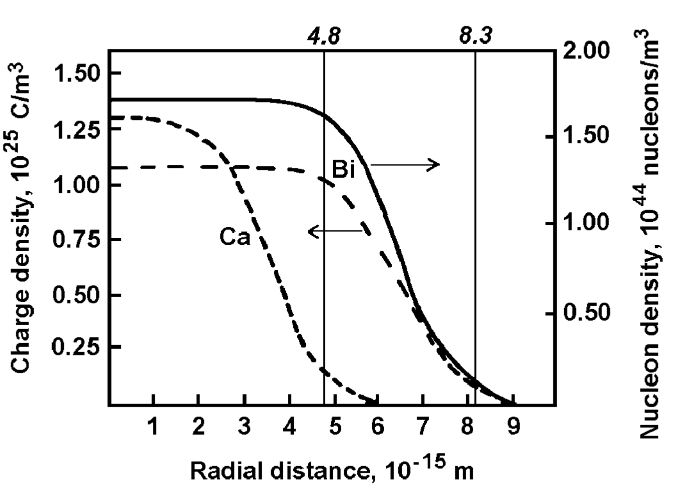


FIG. 3.4. Experimentally measured charge and nuclear density values for $^{40}Ca$ and $^{209}Bi$ as a function of the nuclear radius.

correct, however. By bombarding nuclei with very high energy electrons or protons (up to $\geq$ 1 GeV) and measuring the scattering angle and particle energy, the charge and matter density near the surface of the irradiated nucleus can be studied. These experiments have led to the conclusion that nuclei do not possess a uniform charge or matter distribution out to a sharp boundary, but rather are fuzzy as indicated by the s-shaped curves in Figure 3.4. With an atomic number greater than 20 it has been found that a uniform charge and mass density exists over a short distance from the center of the nucleus, and this core is surrounded by a layer of decreasing density which seems to have a constant thickness of $\sim 2.5$ fm independent of mass number. In a bismuth nucleus, for example, the density remains relatively constant for approximately 5 fm then decreases steadily to one-tenth of that value in the next 2 fm (Fig. 3.4). It has also been found that not all nuclei are spherical, some being oblate and others prolate around the axis of rotation.

Despite the presence of this outer layer of decreasing density and the nonspherical symmetry, for most purposes it is adequate to assume a constant density nucleus with a sharp boundary. Therefore, use is made of the radius equation (3.7) in which the $r_o$ value may be assumed to be 1.4 fm. Using this relationship, we can calculate the radius of $^{40}$Ca to be $r = 1.4 \times 10^{-15} \times 40^{1/3} = 4.79$ fm, and for $^{209}$Bi to be 8.31 fm. These values are indicated in Figure 3.4. For $^{80}$Br a similar calculation yields 6.0 fm, while for $^{238}$U the radius calculated is 8.7 fm. From these calculations we see that the radius does not change dramatically from relatively light nuclei to the heaviest.

### 3.6. Semiempirical mass equation

In preceding sections we have learned that the size as well as the total binding energy of nuclei are proportional to the mass number. These characteristics suggest an analogy between the nucleus and a drop of liquid. In such a drop the molecules interact with their immediate neighbors but not with other molecules more distant. Similarly, a particular nucleon in a nucleus is attracted by nuclear forces only to its adjacent neighbors. Moreover, the volume of the liquid drop is composed of the sum of the volumes of the molecules or atoms present since these are nearly incompressible. Again, as we learned above, this is similar to the behavior of nucleons in a nucleus. Based on the analogy of a nucleus to a droplet of liquid, it has been possible to derive a semiempirical mass equation containing various terms which are related to a nuclear droplet.

Let us consider what we have learned about the characteristics of the nuclear droplet. (a) First, recalling that mass and energy are equivalent, if the total energy of the nucleus is directly proportional to the total number of nucleons there should be a term in the mass equation related to the mass number. (b) Secondly, in the discussion of the neutron/proton ratios we learned that the number of neutrons could not become too large since the discrepancy in the energy levels of the neutron and proton play a role in determining the stability of the nucleus. This implies that the binding energy is reduced by a term which allows for variation in the ratio of the number of protons and neutrons. (c) Since the protons throughout the nucleus experience a mutual repulsion which affects the stability of nucleus, we should expect in the mass equation another negative term reflecting the repulsive forces of the protons. (d) Still another term is required to take into account that the surface nucleons, which are not completely surrounded by other nucleons, would not be totally saturated in their attraction. In a droplet of liquid this lack of saturation of surface forces gives rise to the effect of surface tension. Consequently, the negative term in the mass equation reflecting this unsaturation effect should be similar to a surface tension expression. (e) Finally, we have seen that nuclei with an even number of protons and neutrons are more stable than nuclei with an odd number of either type of nucleon and that the least stable nuclei are those for odd numbers of both neutrons and protons. This odd-even effect must also be included in a mass equation.

Taking into account these various factors, we can write a semiempirical mass equation. However, it is often more useful to write the analogous equation for the mass defect or binding energy of the nucleus, recalling (3.5). Such an equation, first derived by C. F. von Weizsäcker in 1935, would have the form:

$$E_B(\text{MeV}) = a_v A - a_a (N - Z)^2/A - a_c Z^2/A^{1/3} - a_s A^{2/3} \pm a_\delta/A^{3/4} \quad (3.8)$$

The first term in this equation takes into account the proportionality of the energy to the total number of nucleons (the volume energy); the second term, the variations in neutron and proton ratios (the asymmetry energy); the third term, the Coulomb forces of repulsion for protons (the Coulomb energy); the fourth, the surface tension effect (the surface energy). In the fifth term, which accounts for the odd-even effect, a positive sign is used for even proton-even neutron nuclei and a negative sign for odd proton-odd neutron nuclei. For nuclei of odd $A$ (even-odd or odd-even) this term has the value of zero. Comparison of this equation with actual binding energies of nuclei yields a set of coefficients; e.g.

$$a_v = 15.5,\ a_a = 23,\ a_c = 0.72,\ a_s = 16.8,\ a_\delta = 34$$

With these coefficients the binding energy equations (3.2) and (3.5) give agreement within a few percent of the measured values for most nuclei of mass number greater than 40.

When the calculated binding energy is compared with the experimental binding energy, it is seen that for certain values of neutron and proton numbers, the disagreement is more serious. These numbers are related to the so-called "magic numbers", which we have indicated in Figure 3.1, whose recognition led to the development of the nuclear shell model described in a later chapter.

### 3.7. Valley of β-stability

If the semiempirical mass equation is written as a function of $Z$, remembering that $N = A - Z$, it reduces to a quadratic equation of the form

$$E_B = aZ^2 + bZ + c \pm d/A^{3/4} \quad (3.9)$$

where the terms $a$, $b$ and $c$ also contain $A$. This quadratic equation describes a parabola for constant values of $A$. Consequently, we would expect that for any family of isobars (i.e. constant $A$) the masses should fall upon a parabolic curve. Such a curve is shown in Figure 3.5. In returning to Figure 3.1, the isobar line with constant $A$ but varying $Z$ cuts diagonally through the line of stable nuclei. We can picture this as a valley, where the most stable nuclei lie at the bottom of it (cf. Figs. 3.1 and 3.5), while unstable nuclei lie up the valley

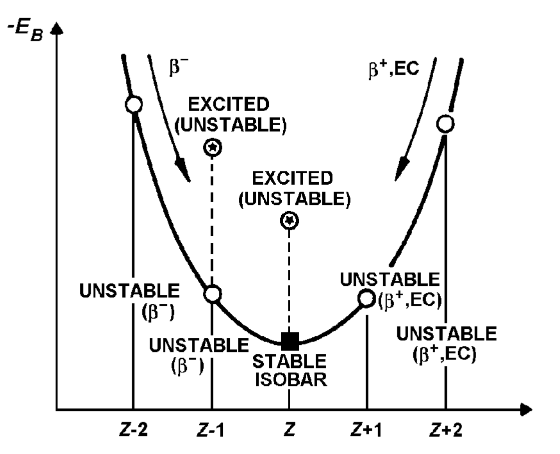


FIG. 3.5. Isobar cut across the valley of stability showing schematically the position of different kinds of nuclei.

sides as shown in Figure 3.5. Any particular isobaric parabola can be considered as a cross-section of the valley of stability; Figure 3.5 would be seen by someone standing up

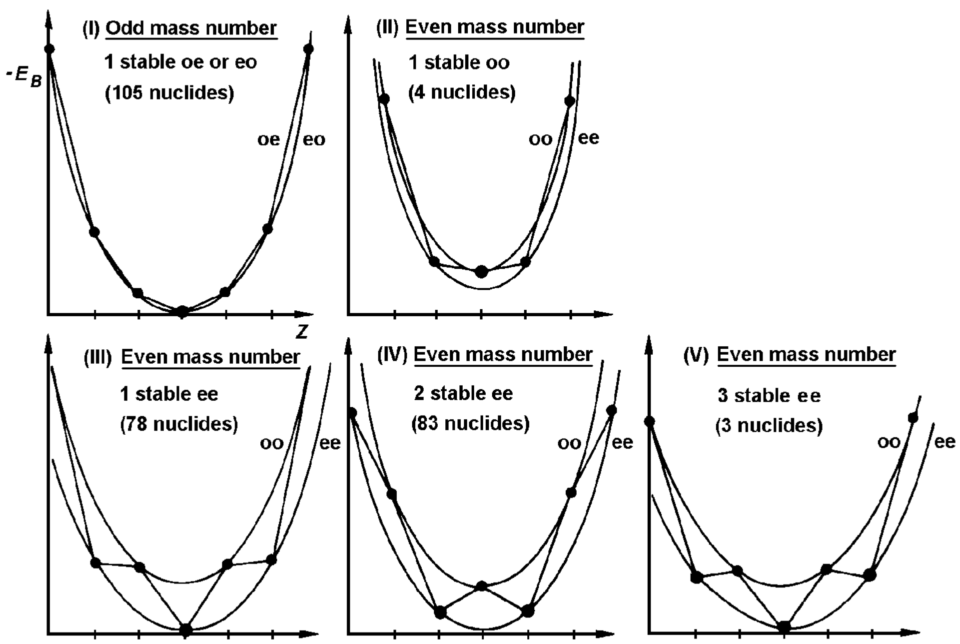


FIG. 3.6. Isobar parabolas for odd mass numbers (I: odd-even or even-odd nuclides) and for even mass numbers (cases II - V). The stable nuclides are indicated by heavier dots.

to the right of Figure 3.1 and looking down the valley. The isobars located on the sides of the parabola (or slope of the valley) are unstable to radioactive decay to more stable nuclides lower on the parabola, though usually the most stable nucleus is not located exactly at the minimum of the parabola. Nuclides on the left hand side of the parabola (lower atomic numbers) are unstable to decay by $\beta^-$emission. Isobars to the right of the valley of stability are unstable to $\beta^+$ decay or electron capture. At the bottom of the valley the isobars are stable against $\beta^-$ decay. The curved line in Figure 3.1 is calculated for maximum stability according to (3.8), and indicates the theoretical bottom of the valley. The minimum of the curve can be calculated from (3.8) to be

$$Z = 2A/[4 + (a_c/a_a)A^{2/3}] \tag{3.10}$$

and is shown in Figure 3.1. For small $A$ values (3.10) reduces to $Z = A/2$ or $N = Z$; thus the bottom of the stability valley follows the $N = Z$ line as indicated in Figure 3.1 for the lighter nuclides.

A closer analysis of (3.9) makes us expect that the last term gives rise to three different isobaric parabola depending on whether the nuclei are odd-*A* (even-odd or odd-even), odd-odd, or even-even (Fig. 3.6). In the first case, in which the mass number is odd, we find a single parabola (I); whether all beta decay leads to changes from odd-even to even-odd, etc. For even mass numbers one finds a double parabola (II) – (V). When the individual nuclear properties are considered, the difference between the curves for the odd-odd and even-even nuclei may lead to alternatives with regard to the numbers of possible stable isobars: it is possible to find three stable isobars (case V) although two (case IV) are more common. Although the odd-odd curve always must lie above the even-even curve, still an odd-odd nucleus may become stable, as is shown for case II.

## 3.8. The missing elements: $_{43}$Tc and $_{61}$Pm

Among the stable elements between $_1$H and $_{82}$Pb two elements are "missing": atomic number 43, named *technetium* (Tc), and atomic number 61, *promethium* (Pm). Though these elements can be produced through nuclear reactions and also have been found to exist in certain stars, they are not found on earth because their longest lived isotopes have much too short half-lives for them to have survived since the formation of our planet. This can be understood by considering the valley of β-stability. For pedagogic reasons we will first discuss promethium.

### 3.8.1. *Promethium*

The valley of β-stability for $Z = 61$ shows a minimum around mass number $A = 146$, for which the isotopes are either of the even-even or of the odd-odd type. Thus the binding energy curveshould exhibit two isobar parabolas, as illustrated in Figure 3.7; the decay energy $Q$ is released binding energy. $^{146}$Pm has a 5.5 y half-life and decays either by electron capture (63%) to $^{148}$Nd or by $\beta^-$-emission (37%) to $^{146}$Sm, who both are more stable (i.e. have a larger nucleon binding energy); the nuclear binding energy is given on

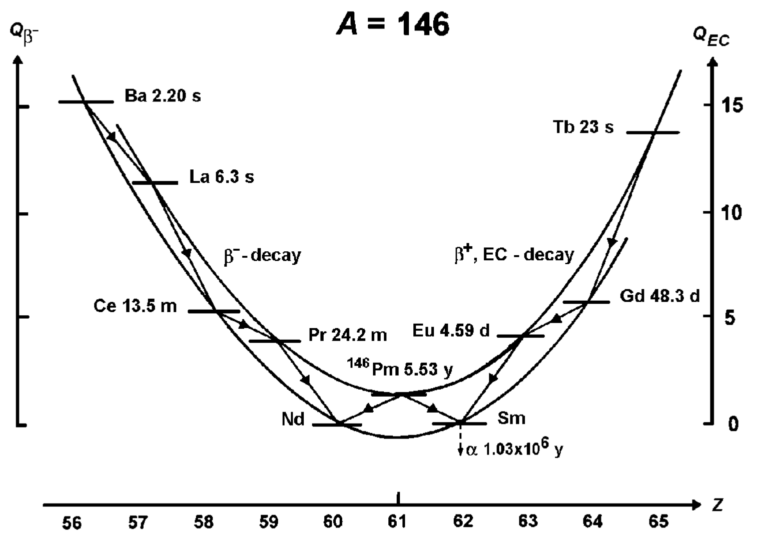


FIG. 3.7. Decay scheme for $A$ = 146, with isobar half-lives. Decay energy $Q$ in MeV. $^{146}$Nd and $^{146}$Sm are β-stable.

the vertical scale in the Figure. The curves shown in Figure 3.7 differ from those calculated from eqn. 3.8 by about 1 unit in Z due to deviations from the simple liquid drop model in the lanthanide region, see $11.4.

The two adjacent isobars, mass numbers $A$ = 145 and $A$ = 147, are of the odd-even and even-odd types, thus only one isobaric β-decay curve exists for each of these. The decay scheme for $A$ = 145 follows curve I in Figure 3.6 for which $^{145}_{60}$Nd is the stable isobar. $^{145}$Pm is the longest lived promethium isotope ($t_{1/2}$ 17.7 y). For $A$ = 147, the stable isobar is $^{147}_{62}$Sm; the half-life of $^{147}$Pm is 2.62 y, which makes it the most convenient radioisotope of promethium for use in experiments.

Promethium is a fission product (Ch. 4 and 19) and can be chemically isolated in pure form. It exhibits typical lanthanide properties and is used in technology and medicine as a radiation source (Ch. 9).

### 3.8.2. *Technetium*

For technetium, $Z$ = 43, the valley of β-stability has a minimum in the neighborhood of $N$ = 55 and thus, for $Z$ = 43, $A$-values around 97 and 99 are most likely to be stable (recall that odd-odd nuclei are less stable than odd-even). If one considers all the isobars between $A$ = 95 and 102 one finds that for each mass number in this range there is already at least one stable nuclide for the elements with $Z$ = 42 (molybdenum) and $Z$ = 44 (ruthenium). Since adjacent isobars cannot both be stable, this excludes the possibility of

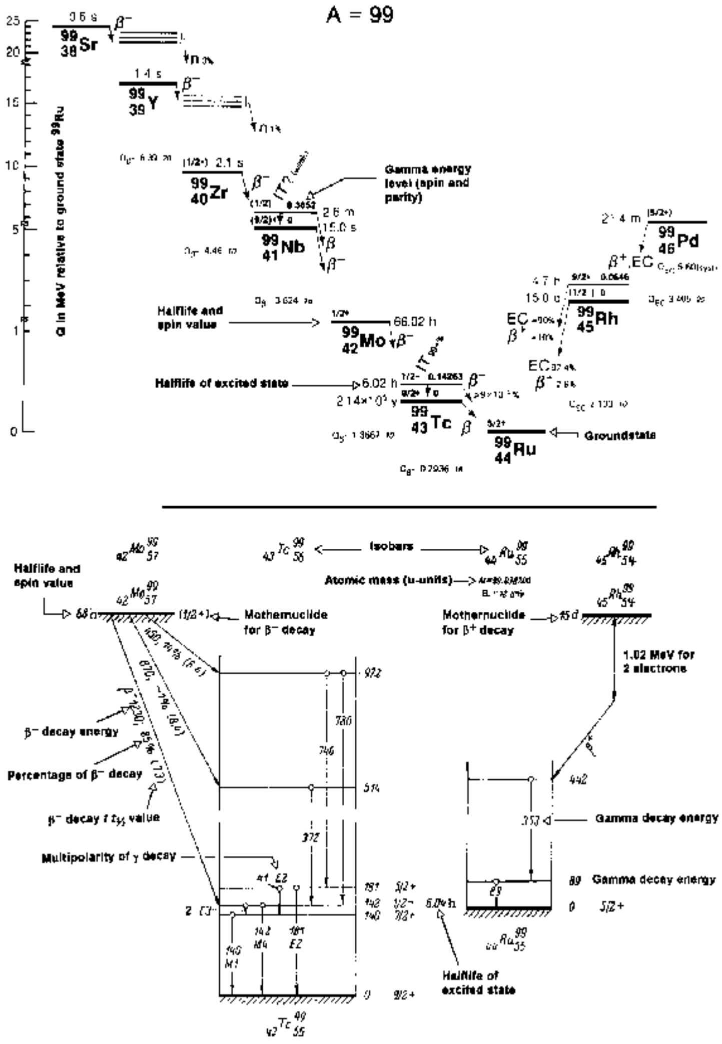


FIG. 3.8. The $A$ = 99 decay scheme (From E. Browne, R. B. Firestone and V. S. Shirley, Table of Radioactive Isotopes, and from B. S. Dzhelepov and L. K. Peker, Decay schemes of radioactive nuclei.).

stable odd-even isotopes of technetium. The longest lived isotopes of technetium are those with $A = 97$ ($2.6 \times 10^6$ y), $A = 98$ ($4.2 \times 10^6$ y), and $A = 99$ ($2.1 \times 10^6$ y). Figure 3.8 shows the decay scheme for $A = 99$, which is taken out of a standard Isotope Table; the vertical axis shows the relative binding energies (broken scale). The Figure illustrates the information normally presented in isotope tables, and will be further explained in subsequent chapters.

Hundreds of kilograms of $^{99}$Tc and its *precursor* (fore-runner) $^{99}$Mo are formed every year as fission products in nuclear reactors, and 10's of kg of Tc have been isolated and studied chemically. Its properties resembles those of its homologs in the Periodic Table - manganese and rhenium. Figure 3.8 shows decay schemes for mass number 99: the upper one from Shirley et al, 1986, the lower one from Dzhelepov et al, 1961; more detailed schemes appear in both references. The ones shown in Figure 3.8 were chosen for pedagogic reasons, and, for this purpose, we have also inserted explanations, some of which will be dealt with later. Older references are often still useful for rapid survey, while the newest ones give the most recent information and refined numerical data.

The upper left part of Figure 3.8 shows a decay chain from fission of $^{235}$U that ends in $^{99}$Ru, the most stable isobar of $A = 99$. The lower diagram shows that the $^{99}$Mo $\beta^-$ decays all reaches the spin/parity ½− level, designated $^{99m}$Tc; this isomer decays with $t_{1/2}$ 6.02 h to long-lived $^{99}$Tc, emitting a single $\gamma$ of 0.142 MeV ($>$ 99%, see upper diagram). The isomer $^{99m}$Tc is a widely used radionuclide in nuclear diagnostics (§9.5), and can be conveniently "milked" from its mother $^{99}$Mo, see §4.16.

## 3.9. Other modes of instability

In this chapter we have stressed nuclear instability to beta decay. However, in §3.4 it was learned that very heavy nuclei are unstable to fission. There is also a possibility of instability to emission of $\alpha$-particles in heavy elements (circles in Figure 3.1) and to neutron and proton emission.

Nuclei are unstable to forms of decay as indicated in Figure 3.1. For example, making a vertical cut at $N = 100$, the instability from the top is first proton emission, then, $\alpha$-emission (for $N = 60$ it would instead be positron emission or electron capture, as these two processes are about equally probable), and, after passing the stable nuclides (the isotones $^{170}$Yb, $^{169}$Tm and $^{168}$Er), $\beta^-$emission and, finally, neutron emission. This is more clearly indicated in Appendix C, and for the heaviest nuclides (i.e. $Z \geq 81$) in Figures 5.1 and 16.1. For $\alpha$-decay the Figure indicates that for $A > 150$ ($Z \geq 70$, $N > 80$) the nuclei are $\alpha$-unstable, but in fact $\alpha$-decay is commonly observed only above $A \approx 200$. This is due to the necessity for the $\alpha$-particle to pass over or penetrate the Coulomb barrier (cf. §11.7.3). Although neutron and proton emissions are possible energetically, they are not commonly observed as the competing $\beta$-decay processes are much faster.

## 3.10. Exercises

**3.1.** Calculate the nucleon binding energy in $^{24}$Mg from the atomic mass excess value in Table 3.1.

**3.2.** How many times larger is the nucleon binding energy in $^{24}$Na than the electron binding energy when the ionization potential of the sodium atom is 5.14 V?

**3.3.** Assuming that in the fission of a uranium atom an energy amount of 200 MeV is released, how far would 1 g of $^{235}U$ drive a car which consumes 1 liter of gasoline (density 0.70 g $cm^{-3}$) for each 10 km? The combustion heat of octane is 5500 kJ $mole^{-1}$, and the combustion engine has an efficiency of 18%.

**3.4.** Estimate if fusion of deuterium into helium releases more or less energy per gram of material consumed than the fission of uranium.

**3.5.** When a neutron is captured in a nucleus, the mass number of the isotope increases one unit. In the following Table mass excess values are given for three important isotope pairs:

| | | | |
|---|---|---|---|
| $^{235}U$ | 40 915 keV | $^{236}U$ | 42 441 keV |
| $^{238}U$ | 47 306 | $^{239}U$ | 50 571 |
| $^{239}Pu$ | 48 585 | $^{240}Pu$ | 50 122 |

If the average nucleon binding energy in this region is 7.57 MeV one can calculate the difference between this average binding energy and the one really observed in the formation of $^{236}U$, $^{239}U$, and $^{240}Pu$. Calculate this difference. Discuss the possible significance of the large differences observed for the $^{238}U/^{239}U$ pair as compared to the other pairs in terms of nuclear power.

**3.6.** With the semiempirical mass equation (3.8) estimate the binding energy per nucleon for $^{10}B$, $^{27}Al$, $^{59}Co$, and $^{236}U$. Compare the results with the observed values in Table 3.1.

**3.7.** With eqn. (3.10) determine the atomic number corresponding to maximum stability for $A = 10, 27, 59$, and 239. Compare these results with the data in the isotope chart, Appendix C.

## 3.11. Literature

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