## CHAPTER 19

# Principles of Nuclear Power

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After the discovery by Hahn, Strassman, Meitner and Frisch in 1938 - 1939 that neutrons induced fission in uranium, and that the number of neutrons released in fission was greater than one, many scientists realized that it should be possible to build a chain-reacting system in which large amounts of nuclear energy were released, *a nuclear reactor*. The first such system was constructed in Chicago in the early 1940s under the scientific leadership of Fermi and became critical on December 2, 1942, as part of the World War II Manhattan Project. Since that time many hundreds of nuclear reactors have been built throughout the world, mostly for power production.



FIG. 19.1. Relative amount of all electricity in each country generated by nuclear power stations during 1999.

In early 2000, 433 nuclear power reactors  $(349 \text{ GW}_e)^1$  were in operation; 120 (108 GW<sub>e</sub>) in North America including the US, 3 (~ 2 GW<sub>e</sub>) in South America, 2 (~ 2 GW<sub>e</sub>) in Africa, 169 (146 GW<sub>e</sub>) in Europe, including the Russian federation, and 90 (66 GW<sub>e</sub>) in the Far East (mainly Japan). Presently 37 power reactors (~ 31 GW<sub>e</sub>) are under construction, the majority in the Far East and most of the rest in Europe. As small old nuclear power plants are shut down and replaced by bigger new ones, the number of reactors may remain constant in a geographic area, or even decrease slightly, although the total installed nuclear generating capacity often continues to increase.

Figure 19.1 shows the fraction of all electricity generated by nuclear power stations in different countries. In countries like Lithuania and France any further large increase in

Reactor type and abbreviation	Number of reactors (%)	Generating capacity (% of nuclear capacity)
Pressurized Water Reactor (PWR)	57.7	63.3
Boiling Water Reactor (BWR)	21.2	22.7
Pressurized Heavy Water Reactor (PHWR)	8.5	6.0
Magnox Reactor (GCR)	4.6	1.3
Pressure Tube Boiling Water Reactor (RBMK)	3.2	4.0
Advanced Gas-cooled Reactor (AGR)	3.2	2.5
Liquid Metal Fast Breeder Reactor (LMFBR)	0.5	0.2
Electricity Generating District Heating Reactor (EGP)	0.9	< 0.1

TABLE 19.1. Power reactor types in common use

Data refer to end of 1999

 $^{1}$  GW<sub>e</sub> = gigawatt electric power installed

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nuclear power production capacity is unlikely as this could lead to a generating capacity in excess of demand. Furthermore, opposition to the peaceful use of nuclear power has led to revised plans for the use of nuclear power in many countries in the West (cf. Ch. 22). In some cases earlier expansion plans have been replaced by moratoria on new plants, by a decision to shut-down some or all operating nuclear power stations (Germany, Italy and Sweden), or by plans to force the utilities to shut down all nuclear power stations at some future date (Germany and Sweden).

Nuclear chemistry plays an essential part in achieving safety and reliability in this source of power. Nuclear chemists and engineers are responsible for much of the nuclear fuel cycle, from uranium ore processing to ultimate disposal of radioactive waste.

## 19.1. The nuclear reactor

The fission process with thermal neutrons can be summarized as follows

$$^{235}\text{U} + \text{n} \rightarrow \text{FP} + \text{vn}$$

For each neutron consumed on the average ~ 2.5 (= v) new neutrons are released. The new neutrons can be used to fission other <sup>235</sup>U nuclei leading to the release of even more neutrons forming a *nuclear chain reacting system*. The kinetic energy of the fission fragments is promptly converted to heat through collisions with other atoms. In a *nuclear reactor* the nuclear chain reactions are controlled so that an equilibrium state is reached, where for each fission exactly one of the new neutrons is used for further fission. Under these conditions the *neutron multiplication factor k* is said to be 1. If the factor is higher the number of neutrons, and consequently the fission rate, increases exponentially. Without any control mechanism the heat evolved would ultimately destroy the chain-reacting system. This is extremely improbable in conventional reactors because of various active and passive control mechanisms. No such control mechanisms are present in *nuclear explosive devices* and, instead, every effort is made to make the reaction as violent as possible.

The neutron transport equation describes the interaction of neutrons with their environment. The accurate design of a chain reacting system requires solution of this equation. The complete solution yields the neutron flux and neutron spectrum as function of the space coordinates and time. As there is no known analytical solution to this equation for a real chain reacting system, numerical methods or various approximations must be used. In this text we do not consider numerical solution methods. We illustrate the general properties of chain reacting systems of interest to the nuclear chemist by using some approximate solutions assuming that the neutron flux and its energy spectrum is known.

There is quite a flexibility in the design of a controlled nuclear chain reacting system. Because each concept has its advantages and drawbacks, more than a dozen different types of nuclear reactors have been developed and tested. However, only a few types are common. About 79% of the currently operating nuclear power plants are of the *light water reactor* type (LWR), i.e. *pressurized water reactors* (PWR)<sup>1</sup> or *boiling water reactors* (BWR); cf. Table 19.1. We therefore begin our discussion of reactors by describing a LWR

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<sup>&</sup>lt;sup>1</sup> Until Ch.20 we will not discuss other existing pressurized water (or boiling water) reactor designs.





FIG. 19.2. Main components of a pressurized light water cooled and -moderated nuclear power reactor (PWR) and a view of the Ringhals plant (Sweden) with 3 PWRs and 1 BWR.

station with emphasis on the principles and main components. Figure 19.2 shows a schematic diagram of a PWR and a picture of operating LWRs, three PWRs and one BWR (highest chimney).

The principal component in any reactor is the *core* which contains the fissionable *fuel* material. This is usually  $UO_2$  enriched in <sup>235</sup>U to 2 – 4% shaped as short cylinders, *pellets*, which are stacked in zirconium alloy tubes (the *can*, *canning* or *cladding*) forming *fuel rods* or *fuel pins*. The fuel rods are mounted together in clusters forming *fuel elements* or *fuel* assemblies.

When fission occurs in the nuclear fuel practically all of the immediately released fission energy occurs as kinetic energy of fission fragments and neutrons (cf. Table 14.1 and §19.2). The kinetic energy is rapidly converted into heat. In order to maintain a stable operating temperature in the reactor it is necessary to provide a *coolant*. The coolant material, which is water in LWRs, enters the core from below, is heated while passing upwards along the fuel rods and leaves the core at the top as either hot water at high pressure (in the *pressurized water reactor*, PWR, as indicated in Figure 19.2) or as a high pressure high temperature steam – water mixture (in the *boiling water reactor*, BWR). The following discussion will concentrate on the operation of a PWR because it is the most common reactor type.

In the PWR steam is generated at lower pressure on the secondary side of separate heat exchangers, *steam generators*. The steam drives one or more turbines connected to electric generators and is condensed to water by an *external cooling system* after the turbine. Except for the external cooling system (sea or river water, cooling towers, etc.), the steam-water flow systems are closed cycles.

The neutron flux, and hence the rate of the fission chain reaction, in the reactor core is controlled by movable *control rods* which contain material with high cross sections for absorption of neutrons and, in a PWR, also by an absorber dissolved in the coolant.

In order to maximize the cross section for fission, which is greatest for low energy neutrons, the neutrons are slowed down or "moderated" by a material (the *moderator*) that elastically scatters neutrons but has a small neutron capture cross section. In LWRs ordinary (but very pure) water serves the purpose of both moderation and cooling (in other reactor types the moderator may be a liquid like  $D_2O$ , a solid material like graphite or absent and the cooling medium may be a gas or a metal like lead, mercury or sodium).

Reactors of this kind, in which the fuel is physically separated from the moderator, are said to be of the *heterogeneous type*, while in *homogeneous reactors* the fuel is directly dissolved in the moderator material. Homogeneous reactors have only been built for experimental purposes.

The reactor core system is enclosed in a stainless steel clad pressure vessel of high strength steel, thermally insulated on the outside. In order to protect the operating personnel against hazards from the neutrons and  $\gamma$ -rays emitted in fission, the pressure vessel is surrounded by a thick *biological shield*. The reactor system and steam generators are normally enclosed in a *containment vessel*. In case of a potentially dangerous malfunction in the reactor the containment is sealed off, thus presenting a barrier against escape of radioactive material to the surroundings. Air and water effluents from the reactor station during normal operation are monitored and (when prescribed activity levels are exceeded) purified from radioactive contaminants.

The fuel elements can only be used to much less than 100% consumption of the  $^{235}$ U originally present. Fission leads to the production of fission products, which accumulate in the fuel. Some of the fission products have very high neutron capture cross sections (> 100 b) and compete with the fission chain reaction for the neutrons. Before the reactor becomes "poisoned" by these fission products, part of the fuel elements has to be replaced. In case of LWR:s the reactor is shut down, the top of the pressure vessel removed and some of the older fuel elements replaced by new ones. The replacement is done by means of a fuel *charging* (discharging) *machine* (a few other reactor types permit fuel replacement during operation). Because of the large amounts of radioactive fission products, the used fuel elements are always allowed to "cool" (with respect to both radioactivity and heat) for months, often years, in water-filled *storage pools* located inside the containment (see Fig. 19.16).

The used fuel elements may later be *reprocessed* to recover the remaining amount of fissile material as well as any *fertile material* or regarded as waste; fertile atoms are those which can be transformed into fissile ones, i.e. <sup>232</sup>Th and <sup>238</sup>U, which through neutron capture and  $\beta$ -decays form fissile <sup>233</sup>U and <sup>239</sup>Pu, respectively. The chemical reprocessing removes the fission products and actinides other than U and Pu. Some of the removed elements might be valuable enough to be isolated although this is seldom done. The mixed fission products and waste actinides are stored as *radioactive waste*. The recovered fissile materials may be refabricated (the U may require re-enrichment) into new elements for reuse. This "back-end" of the nuclear fuel cycle is discussed in Chapter 21.

## 19.2. Energy release in fission

From Table 14.1 it is seen that in thermal neutron fission of  $^{235}$ U the fission fragments are released with a kinetic energy of ~ 165 MeV (on the average), the 2.5 *prompt neutrons* have an average kinetic energy of ~ 5 MeV together, and the *prompt*  $\gamma$ *-rays* have an average of 7 MeV. This *prompt energy release* of ~ 177 MeV is absorbed in the surrounding material.

The fission products are radioactive and decay through emission of  $\beta^-$ ,  $\gamma$ , and X-rays; their total amount of decay energy is ~ 23 MeV. About 10 MeV (the value is uncertain) escapes the reactor as radiation, and ~ 1 MeV of the decay energy remains as undecayed fission products in the spent fuel when unloaded from the reactor; thus ~ 12 MeV  $\beta\gamma$  decay energy (divided about equally between  $\bar{E}_{\beta}$  and  $E_{\gamma}$ ) is absorbed in the reactor. The neutrons not consumed by fission are captured in the reactor material with release of binding energy; it is estimated that this amounts to about 10 MeV. Thus the total amount of energy expended per fission in a shielded controlled reactor is about 177 + 12 + 10 ~ 199 MeV. The total energy release varies only a few MeV between the different fissile nuclei. As a practical average value 200 MeV per fission can be used regardless of the fissioning nucleus:

$$E_{\rm f} = 200 \text{ MeV per fission} = 3.20 \times 10^{-11} \text{ J per fission}$$
 (19.1)

Thus  $(3.20 \times 10^{-11})^{-1} \approx 3.1 \times 10^{10}$  fissions s<sup>-1</sup> correspond to the production of 1 W of reactor heat. The heat power of a reactor can be written



FIG. 19.3. Cross sections for n-capture  $(\sigma_{n,\gamma})$ , fission  $(\sigma_f)$ , and total  $(\sigma_{tot})$  as a function of neutron energy.

$$P = E_{\rm f} \,\mathrm{d}N_{\rm f}/\mathrm{d}t \tag{19.2}$$

where  $dN_f/dt$  is the number of fissions per second. A nuclear power station producing 3 GW heat (GW<sub>th</sub>; th for thermal) has an electric output of 1 GW<sub>e</sub> at a 33% *efficiency* in converting the thermal energy into electric; this corresponds to  $8.1 \times 10^{24}$  fissions d<sup>-1</sup>. Since the weight of a <sup>235</sup>U atom is  $M/N_A = 3.90 \times 10^{-25}$  kg, this would correspond to the fission of  $8.1 \times 10^{24} \times 3.9 \times 10^{-25} = 3.16$  kg <sup>235</sup>U d<sup>-1</sup>; the real consumption of <sup>235</sup>U is slightly different and varies with time as discussed in §19.9.

## 19.3. Fission probability

When uranium is irradiated by neutrons, in addition to neutron capture followed by fission (n, f), several different processes occur: scattering,  $(n, \gamma)$ , (n, 2n) reactions, etc. All these reactions are of importance for the reactor designer as well as for the chemists who have to manufacture new fuel pellets and take care of the spent fuel elements. The probability for the various reactions depends on the neutron energy. As is seen in Figure 19.3, three regions are clearly distinguishable:

(1) For thermal neutrons with average kinetic energies  $(\bar{E}_n) \le 1 \text{ eV}$ , fission of  $^{235}\text{U}$  and  $^{239}\text{Pu}$  dominates over neutron capture (i.e.  $\sigma_f > \sigma_{n,\gamma}$ ). Though  $\sigma_{n,\gamma}$  for  $^{238}\text{U}$  is small considerable capture occurs because of the large fraction of  $^{238}\text{U}$  present.

(2) For epithermal neutrons  $(1 \le \overline{E}_n \le 10^5 \text{ eV})$  large radiative capture and fission resonances occur. In this region heavier isotopes are formed by  $(n,\gamma)$  reactions:  ${}^{235}U(n,\gamma){}^{236}U$ ;  ${}^{238}U(n,\gamma){}^{239}U$ ;  ${}^{239}Pu(n,\gamma){}^{240}Pu$ ;  ${}^{240}Pu(n,\gamma){}^{241}Pu$ ; etc (see Fig. 19.5).



FIG. 19.4. Neutron spectra of some chain reacting systems and a proposed accelerator driven system. The abscissa shows neutron flux times energy on an arbitrary scale.

(3) For fast neutrons ( $\bar{E}_n \ge 0.1$  MeV) the cross sections are relatively small,  $\le 1$  b. Fission dominates over radiative capture. Of particular importance is that <sup>238</sup>U becomes fissionable at a neutron energy of ~ 0.6 MeV; its fission cross section increases with neutron energy above the threshold to a constant value of ~ 0.5 b at  $\ge 2$  MeV.

It is obvious that the neutron energy spectrum of a reactor plays an essential role. Figure 19.4 shows the prompt (unmoderated) fission neutron spectrum with  $\bar{E}_n \sim 2$  MeV. In a nuclear explosive device almost all fission is caused by fast neutrons. Nuclear reactors can be designed so that fission mainly occurs with fast neutrons or with slow neutrons (by moderating the neutrons to thermal energies before they encounter fuel). This leads to two different reactor concepts – the *fast reactor* and the *thermal reactor*. The approximate neutron spectra for both reactor types are shown in Figure 19.4. Because thermal reactors are more important at present, we discuss this type of reactors first.

In an ideal thermal reactor we may assume that most of the neutrons are in thermal equilibrium with the moderator atoms, though in practice the neutron spectrum in power reactors is much more energetic due to incomplete moderation. From  $\bar{E}_n = \mathbf{k}T$  it follows that at about 300°C (a typical moderator temperature in a LWR)  $\bar{E}_n$  is 0.05 eV; cf. Fig. 19.4. However, "thermal neutron energy" ( $E_{th}$ ) cross sections are standardized to mono-energetic neutrons with a velocity of 2200 m s<sup>-1</sup>, which corresponds to  $E_n$  0.025 eV according to (2.22). Cross sections at this energy are given in Table 19.2. For most (but not all) nuclides the cross section at neutron energies below the resonance region may be estimated by the 1/v law (§14.4).Because thermal reactors have neutrons of more than one energy, effective cross sections<sup>1</sup> must be used for calculating reactor product yields. Such

<sup>&</sup>lt;sup>1</sup> The effective cross-section is given by  $\sigma_{\text{eff}} = \int \sigma(E) \phi(E) dE / \int \phi(E) dE$ , where  $\phi$  is flux and *E* neutron energy.



FIG. 19.5. Nuclear reactions in irradiation of uranium. Figures along the arrows are half lives or effective cross-sections (b) for a typical power LWR, with thermal (0.025 eV) data within parentheses.

cross sections are usually calculated as an average over the core and vary with time as well as from reactor to reactor. Typical LWR effective cross sections are given in Table 19.2 and Figure 19.5 for thin targets. Self screening (depression of the n-flux due to a high  $\Sigma_a$ ) reduces the effective cross sections below those given in Table 19.2 and Figure 19.5 for isotopes with high concentration in the fuel pins.

## 19.4. The fission factor

In thermal reactors <sup>235</sup>U is consumed mainly by fission and radiative capture

 $^{236}U$  decays through  $\alpha$ -emission. Due to its long half-life  $(2.3\times10^7~y)$  it accumulates in the reactor.  $^{236}U$  may capture a neutron, forming  $^{237}U$ , which over a few days time decays to  $^{237}Np$ . The *neutron yield per fission*,  $\nu$ , is an average value which depends on the neutron

	$^{232}\mathrm{Th}$	$^{233}$ U	$^{235}\mathrm{U}$	$^{238}$ U	Nat. U	$^{239}\mathrm{Pu}$	$^{240}\mathrm{Pu}$	$^{241}\mathrm{Pu}$
Radioactive decay Half-life (years) Specific rad. act. (GBq/kg) Specific decay heat (W/kg)	$\alpha, sf$ 1.405×10 <sup>10</sup> 4.06×10 <sup>-3</sup> 0.003	$^{\alpha}_{1.592 \times 10^{5}}_{356}$	$\frac{\alpha}{7.038 \times 10^8}$ 0.0800 0.056	α,sf 4.466×10 <sup>9</sup> 0.01244 0.008	1 1 1 1	α,sf 2.411×10 <sup>4</sup> 2296 1876	α,sf 6.563×10 <sup>3</sup> 8398 6950	$\alpha, sf$ 14.35 3.83×10 <sup>6</sup> 3.250
Thermal neutrons (0.025 eV) n, $\gamma$ -capture ( $\sigma_{\gamma}$ barns) fission ( $\sigma_{f}$ barns) neutron yield (v) fission factor ( $\eta$ )	7.40 39 µb	47.7 531.1 2.492 2.287	98.6 582.2 2.418 2.068	2.70 < 0.5 mb	3.39 4.19 2.418 1.34	268.8 742.5 2.871 2.108	289.5 0.030 2.90 ? 0.0003 ?	368 1009 2.927 2.145
Fast neutrons ( $\sim 0.25$ ; $\sim 1.0$ n, $\gamma$ -capture ( $\sigma_{\gamma}$ barns) fission ( $\sigma_{f}$ barns) neutron yield (v) fission factor ( $\eta$ )	MeV) 0.18; 0.12 ; 0.067 ; 2.1 ; 0.75	0.22; 0.056 2.20; 1.90 2.503; 2.595 2.28; 2.52	0.28; 0.11 1.28; 1.19 2.426; 2.522 1.99; 2.31	0.15; 0.14 ; 0.018 ; 2.50 ; 0.28	0.15; 0.14 0.01; 0.03 2.426; 2.50 0.15; 0.44	0.17; 0.04 1.53; 1.73 2.886; 3.001 2.60; 2.93	0.15; 0.18 0.10; 1.06 2.80; 2.95 1.13; 2.52	2.0; 2.0 1.78; 1.56 3.01; 3.12 1.4; 1.4
Reactor spectrum neutrons ( ~ n, y-capture ( o <sub>Y</sub> barns) fission ( o <sub>f</sub> barns) neutron yield (v) fission factor (ŋ)	· PWR; LMFBR <sup>†</sup> ) 4; 0.30 ; 0.01 ; 2.4 ; 0.08	28; 0.23 298; 2.52 2.49; 2.6 2.27; 2.4	107.5; 0.584 448.1; 1.951 2.43; 2.445 1.96; 1.882	1.5; 0.271 ; 0.040 ; 2.900 ; 0.373		<b>559.5</b> ; 0.547 <b>104</b> 8.7; 1.810 <b>2.87</b> ; <b>2.922</b> <b>1.86</b> ; <b>2.244</b>	2616.8; 0.523 ; 0.371 ; 2.976 ; 1.235	428.7; 0.568 1138.6; 2.640 3.06; 2.968 2.22; 2.443

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energy; it is 2.42 for thermal fission of  $^{235}$ U (see Table 19.2). If we define the ratio

$$\alpha = \sigma_{v} / \sigma_{f} = \Sigma_{v} / \Sigma_{f}$$
(19.4)

(cf. (14.6)) the probability that the captured neutron gives rise to fission is  $\sigma_f/(\sigma_f + \sigma_\gamma) = 1/(1 + \alpha)$ . The number of neutrons produced for each neutron captured is

$$\eta = \nu / (1 + \alpha) \tag{19.5}$$

where  $\eta$  is the *neutron yield per absorption*, also called the *fission factor*. A primary requirement for a chain reaction is that  $\eta > 1$ . Table 19.2 contains  $\eta$ -values for the most important nuclides. At thermal energies  $\eta$  is highest for <sup>233</sup>U, while for fast neutrons  $\eta$  is highest for <sup>239</sup>Pu.

In mixtures of isotopes the macroscopic cross section  $\Sigma$  must be used in calculating  $\alpha$ . Since for natural uranium  $\Sigma_f = 0.72 \times 582$  per 100 and  $\Sigma_{\gamma} = (0.72 \times 99 + 99.3 \times 2.70)$  per 100, one obtains  $\alpha = 0.81$  and  $\eta = 1.34$ , which means a chain reaction is possible with thermal neutrons in natural uranium.

#### 19.5. Neutron moderation

The mode of moderation of the neutrons is one of the crucial design features of a thermal reactor. The fast fission neutrons lose their kinetic energy through elastic scattering with the atoms of the moderator and construction material. In §12.5 the equations are given for the energy change in such collision processes, but only for head-on collisions. Because most collisions involve angular scattering, the number of collisions required to reduce a fast neutron to thermal energy is larger. The *average logarithmic energy decrement* is given by

$$\xi = \ln(E_n/E_n) = 1 + [(A-1)^2/(2A)] \ln[(A-1)/(A+1)]$$
(19.6)

where  $E_n$  is the neutron energy after collision with a moderator atom of mass number *A*. From Table 19.3 it is seen that "light" water (i.e. H<sub>2</sub>O) is most effective in reducing the neutron velocity. The average number of collisions *n* required to reduce the neutron energy from  $E_n^0$  to  $E_n$  is given by

$$n = \xi^{-1} \ln(E_{\rm n}^0/E_{\rm n}) + 1 \tag{19.7}$$

The *slowing down power* (*SDP*) of a moderator depends in addition on the neutron scattering cross section and number of scattering atoms per unit volume ( $N_0$ ):

$$SDP = \xi N_0 \sigma_s = \xi \Sigma_s \tag{19.8}$$

The *SDP* is an average value over the epithermal neutron energy region. A good neutron moderator should divert few neutrons from the fission process, i.e. the neutron absorption cross section must be small. In this respect both heavy water (i.e.  $D_2O$ ) and carbon are

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Property	H <sub>2</sub> O	$D_2O$	Be	С
$N_0 \times 10^{-30}$ (atoms m <sup>-3</sup> )	0.0333	0.0332	0.123	0.0837
$\rho$ (kg m <sup>-3</sup> )	997	1104	1848	1670
$\sigma_{a}$ (th) (barns)	0.66	0.0013	0.0076	0.0035
$\Sigma_{a}^{a}(\text{th}) (\text{m}^{-1})$	2.2	0.0044	0.094	0.029
$\sigma_{e}$ (epith) (barns)	49	10.6	5.9	4.7
ξ	0.927	0.510	0.206	0.158
$\xi \times \Sigma_{\rm s}({\rm epith})/\Sigma_{\rm a}({\rm th})$	68	4095	160	211
$L_{\rm m}^2 ({\rm m}^2)$	0.000713	1.51	0.043	0.287
$\tau$ (m <sup>2</sup> )	0.0031	0.0125	0.0097	0.0349

TABLE 19.3. Physical constants of some moderator materials

superior to  $H_2O$ , since for  $H_2O$  the reaction probability for  ${}^{1}H(n,\gamma){}^{2}H$  is relatively large (Table 19.3). In order to incorporate this property, the concept *moderating ratio* (*MR*) is used as a criterion of the moderator property in the thermal neutron energy region:

$$MR = SDP/\Sigma_a = \xi \Sigma_s / \Sigma_a \tag{19.9}$$

The moderating properties are summarized in Table 19.3. The moderator qualities decrease in the order  $D_2O > C > Be > H_2O$ . In a commercial reactor, price and other properties must also be considered, so  $H_2O$  is favored over  $D_2O$  in most reactors.

#### 19.6. The neutron cycle

In order for a chain reaction to occur at least one of the neutrons released in fission must produce a new fission event. This condition is defined by the *multiplication factor k*:

when k > 1 the number of neutrons in the second generation exceeds the number of neutrons consumed. Under this condition, the neutron flux, and, thus, the number of fission events increase for each successive neutron generation with a resultant increase in the power production of the reactor (§19.8 discusses further the *reactor kinetics*). If k = 1 the number of fissions per unit time, and thus the energy production, is constant. For k < 1 the chain reaction cannot be maintained. A reactor operating with k = 1 is said to be *critical*, k > 1 *supercritical*, and k < 1 *subcritical*. k is regulated by means of the control rods and in PWRs also by the n-absorber concentration in the water (usually boric acid).

In any neutron generation the neutrons experience a variety of fates (Fig. 19.6). Some neutrons escape the reactor and some are absorbed in the reactor structural materials and shielding (i.e. fuel cans, control rods, moderator, coolant, etc.). To take this loss into account, two different multiplication factors are used:  $k_{\infty}$  refers to a reactor of infinite dimensions (i.e. no leakage) while  $k_{\text{eff}}$  refers to a reactor of a finite size:



FIG. 19.6. The neutron cycle in a thermal reactor. The number of neutrons for 100 in first generation are given in circles (data from an old Belgian research reactor, BR1).

$$k_{\rm eff} = k_{\infty} \Lambda \tag{19.11}$$

A is the fraction of neutrons which are *not* lost through leakage to the surroundings (*the non-leakage factor*). In order to minimize the neutron leakage, the reactor core is surrounded with a *neutron reflector* which for thermal neutrons in LWRs is water (graphite or beryllium are sometimes used in other reactor designs); for fast neutron reflection iron is frequently used.

Let us consider the neutron cycle in a reactor, i.e. the fate of the neutrons as they proceed from one generation to the next (Figure 19.6, where leakage, treated in §19.7, also is included). We start by assuming that we have an air cooled reactor containing natural uranium with a graphite moderator in a configuration which allows negligible leakage ( $\Lambda = 1$ ). If 100 neutrons ( $N_1$ ) are captured by the uranium fuel, the fission releases  $N_1 \eta$  or  $100 \times 1.34 = 134$  new neutrons. The fast neutrons may also cause some fission in nearby uranium atoms, thereby releasing more neutrons. This is measured by  $\epsilon$ , the fast fission factor. Its value in a heterogeneous nuclear reactor depends to a large extent on the moderator and on the dimensions within a fuel element. For natural uranium in graphite  $\epsilon$  is about 1.03, so that the 134 neutrons released in fission by the original 100 thermal neutrons are increased to  $N_1 \eta \epsilon$  (or  $1.03 \times 134 =$ 138) neutrons. In homogeneous reactors  $\epsilon = 1$  because the fission neutrons do not have to pass through any zone with concentrated fuel to reach the moderator.

These energetic neutrons are slowed down through collisions with surrounding atoms and decrease steadily in energy until thermal energies are reached. In the energy range 1 to 10<sup>5</sup> eV, *the epithermal region*, <sup>235</sup>U and <sup>238</sup>U have large resonance peaks for radiative capture, while fission in <sup>235</sup>U dominates at thermal energies (Fig. 19.3). In order to maximize the fission probability, the losses through radiative capture in the epithermal region must be minimized. This can be achieved by physically separating moderator and fuel. The fast

General: 640 MW<sub>e</sub> (1912 MW<sub>th</sub>) BWR plant at Würgassen, Germany, in commercial operation 1975-1995.

*Reactor physics*:  $\theta 4.5 \times 10^{-5}$  (s),  $\eta 1.740$ , *f* 0.865, *p* 0.811,  $\epsilon 1.041$ ,  $k_{\infty} 1.270$ ,  $k_{eff} 1.255$ ,  $L^2 3.67$  (cm<sup>2</sup>),  $\tau 34.83$  (cm<sup>2</sup>),  $B^2 2.32 \times 10^{-4}$  (cm<sup>-2</sup>),  $\phi_{th,average} 4.4 \times 10^{13}$  (cm<sup>-2</sup> s<sup>-1</sup>),  $\phi_{fast,average} 1.9 \times 10^{14}$  (cm<sup>-2</sup> s<sup>-1</sup>),  $\Delta k_{max} 25.5\%$ ,  $\Delta k/k - 2.2 \times 10^{-3}\%/^{\circ}$ C.

Core: 86 600 kg U, 2.6%  $^{235}$ U; critical mass 38.5 kg  $^{235}$ U; burn-up 27 500 MWd t $^{-1}$ ; refueling 1/5 of core annually; storage pool for 125 t fuel; core volume 38 m<sup>3</sup>.

*Fuel element:*  $UO_2$  pellets, 1.43 cm diameter; 49 rods per assembly; 444 assemblies in core; enrichment 2.6% <sup>235</sup>U;  $Gd_2O_3$  burnable poison; zircaloy cladding 0.8 mm.

*Heat and coolant data*: Max. fuel temp. 2580 °C, max. can temp. 370 °C; coolant inlet temp. 190 °C, outlet 285 °C, pressure 7 MPa (70 bar); 280 t coolant,  $2 \times 5200 \text{ m}^3 \text{ h}^{-1}$  internal pump capacity; condenser cooling water 95 000 t  $\text{h}^{-1}$  (River Weser).

neutrons, which easily escape from thin fuel pins, are mainly slowed in the surrounding moderator. Because the moderator is of a material with light atoms, the number of collisions required to slow these neutrons is small. Therefore, it is unlikely that the neutrons encounter a fuel pin (and are captured) until thermal energies have been reached. The probability that the neutrons pass through the energy region of the resonance peaks without capture is called the *resonance escape probability* and denoted by *p*. From the original  $N_1$  neutrons at this point there would be  $N_1 \eta \in p$  neutrons. In a reactor of natural uranium and graphite, *p* is usually around 0.9. Therefore, from the original 100 neutrons  $138 \times 0.9$  or 124 second generation neutrons reach the thermal energy range.

The cross sections for neutron capture increase for all atoms for thermal energy neutrons. As a result, even though low cross section materials are used some neutrons are captured by the structural and moderator materials. The probability for the non-capture of thermal neutrons in this fashion is signified by *f*, *the thermal utilization factor*, which in our case can be assumed to be  $\sim 0.9$ . Thus of the original  $N_1$  neutrons 112 thermal neutrons remain in the second generation to cause fission in the nuclear fuel.

These 112 thermal neutrons constitute the second generation of neutrons which according to our definition of neutron multiplication factor is  $k_{\infty}$  so that

$$k_{\infty} = \eta \in pf \tag{19.12}$$

This relation is known as the *four factor formula*. In our case  $k_{\infty} = 1.12$ .

The value of  $\eta$  depends on how much fissile material the fuel contains. The other three factors have a more complicated dependence of the reactor design including the fuel/moderator ratio, their amounts and shape; some values are given for a practical case in Table 19.4.

For simplicity, let us for a moment regard a homogeneous reactor. For such a reactor

$$f = \Sigma_a (\text{fuel}) / [\Sigma_a (\text{fuel}) + \Sigma_a (\text{mod}) + \Sigma_a (\text{other})]$$
(19.13)

where the denominator is the macroscopic cross section of all absorption reactions in the reactor; the last term includes absorption by impurities in construction materials and by products formed in the course of the operation. When e.g. control rods are inserted  $\Sigma_a$  (other) increases. Effective cross sections must be used for all nuclides.

In order to allow for a decrease in *f* during operation, commercial reactors are designed with  $k_{\infty}$  1.2 – 1.3, rather than the 1.12 given in the example above. Even higher  $k_{\infty}$  values occur as is the case for reactors with highly enriched fuel. In reactors with large  $k_{\infty}$ , which permits high burnup values, a small amount of suitable n-absorbers (*poisons*) may be included in the fuel at the start to decrease  $k_{\infty}$ . Such a poison is gadolinium, especially <sup>157</sup>Gd, which occurs to 16% in natural gadolinium and has a thermal neutron cross section of 254 000 b. Even a small amount of gadolinium considerably decreases *f* (see (19.13)). Although <sup>157</sup>Gd is continuously destroyed ("burned") by n,  $\gamma$ -reactions which convert it to <sup>158</sup>Gd ( $\sigma_{n,\gamma}$  2.5 b) during operation of the reactor, at the same time fission product poisons are formed and fissile nuclei used-up resulting in a compensating effect, which maintains an acceptable value of  $k_{\infty}$ . Another possibility to regulate *k*, mainly used in PWRs, is the addition of a suitable amount of a water soluble n-absorber to the moderator - coolant, *chemical shim control*. When burnup increases, the concentration of the soluble absorber is gradually reduced in order to compensate the poisoning from the fission products and the consumption of fissile atoms.

Because many elements contain isotopes with high neutron capture cross sections for thermal neutrons all materials in the reactor must be extremely pure in order to keep  $k_{\infty}$  as large as possible.

An important factor is the temperature dependence of *k*. A temperature increase usually has little effect on  $\eta \epsilon$ , but f usually increases (i.e. becomes close to 1) because of decreasing moderator density and average neutron energy increase; this causes more neutrons to be captured in  $^{239}$ Pu, which has a low energy (0.3 eV) fission resonance. On the other hand, p decreases because thermal vibrations of atoms cause a doppler broadening of the n, y resonances in the epithermal region, mostly for <sup>238</sup>U. UO<sub>2</sub> fuel has a relatively poor heat conductivity causing the center temperature in the fuel pins to vary strongly with the reactor power level, cf. Ch. 21. This gives the doppler broadening of resonances in the fuel a high fission rate dependence. The doppler broadening also increases capture in the control rods. These latter effects normally dominate over the increase in *f*, so that  $k_{\infty}$  decreases slightly with increasing reactor power and temperature. This is referred to as a negative temperature coefficient. In a BWR a negative temperature coefficient is also caused by the bubbles of steam (void) occurring in the moderator, which reduces the thermalization of neutrons while increasing neutron leakage rate. In BWRs the effect of the void normally dominates over doppler broadening in limiting the reactor power. As the reactor tends to keep the void fraction and fuel temperature constant, the power level of a BWR (and to a smaller degree also of a PWR, where mostly the average fuel temperature is affected) can be adjusted by varying the speed of the circulation pumps at a constant control rod setting.

These are important safety features of a reactor. If *k* increases, so do the fission rate and temperature. If the temperature coefficient was positive, *k* would increase further, leading to further temperature increase, etc. However, with a negative temperature coefficient the reactor controls itself: an increase in power (and thus temperature) decreases *k*, which tends to limit the power increase and vice versa. The temperature coefficient is usually given in  $\%\Delta k/k$  per °C.

#### 19.7. Neutron leakage and critical size

All practical reactors have some leakage of neutrons out of the reactor core. This leakage  $\Lambda$  is approximately described by the fast and thermal leakage factors  $\Lambda_f$  and  $\Lambda_{th}$  respectively:

$$\Lambda = \Lambda_{\rm f} \Lambda_{\rm th} \tag{19.14}$$

The effect of this leakage is included in Figure 19.6, and results in the number of neutrons in the second generation being reduced, from the 112 calculated previously for  $k_{\infty}$ , to  $N_2 = 101$ . Thus  $k_{\text{eff}} = 1.01$  and  $\Lambda = 0.9$ , i.e. 10% of the neutrons are lost through leakage. The example in Figure 19.6 refers to a small air cooled and graphite moderated natural uranium research reactor. In nuclear power stations  $k_{\infty}$  is much larger due to the large  $\eta$ -values (because of the enriched fuel needed to sustain operation for long periods at high power between refuelings), while the leakage is much smaller,  $\Lambda \ge 0.97$  (e.g. see Table 19.4).

The leakage is a function of the geometrical arrangement of reactor core and reflector and of the average distance the neutron travels after formation until it causes a new fission:

$$\Lambda_{\rm f} = e^{-B^2\tau} \tag{19.15}$$

$$\Lambda_{\rm th} = (1 + B^2 L^2)^{-1} \tag{19.16}$$

where  $B^2$  is the geometrical buckling, L the thermal diffusion length, and  $\tau$  is referred to as the *neutron* (or *Fermi*) age.

Combining these equations gives the critical equation:

$$k_{\rm eff} = k_{\infty} e^{-B^2 \tau} / (1 + B^2 L^2) \approx k_{\infty} / (1 + B^2 (L^2 + \tau))$$
(19.17)

The quantity  $L^2 + \tau$  is usually denoted by  $M^2$  and is known as the *migration area* and *M* as the *migration length*:

$$M^2 = L^2 + \tau \tag{19.18}$$

Hence for a large thermal critical reactor one can write

$$k_{\rm eff} = k_{\infty} / (1 + B^2 M^2) = 1$$
(19.19)

The thermal diffusion length *L* is calculated from

$$L^2 = L_{\rm m}^2 \left(1 - f\right) \tag{19.20}$$

where  $L_{\rm m}$  is the diffusion length in the pure moderator.  $L_{\rm m}^2$  and  $\tau$  are given in Table 19.3 for various moderator materials.

The geometrical buckling  $B^2$  depends on the neutron flux distribution in the reactor. This distribution in turn depends on the general geometry of the assembly, including boundary

conditions. The calculation of the buckling for heterogeneous reactors is quite complicated but for homogeneous and bare (no reflector) reactors the following approximate simple relations hold:

$B^2$	$= \pi^2 r^{-2}$	(sphere)	(19.21a)
	$= 3 \pi^2 a^{-2}$	(cube)	(19.21b)
	$= (\pi/h)^2 + (2.405/r)^2$	(cylinder)	(19.21c)
	$= 33 h^{-2}$	(cylinder; height = diameter)	(19.21d)

where *r* is the radius of the sphere and cylinder, *a* the side of the cube, and *h* the cylinder height (which is also the diameter of the cylinder in eqn. 19.21d).

It is seen that  $k_{\text{eff}}$  in (19.17) increases with decreasing buckling, and, because  $B \propto 1/r$  (19.21a), with increasing size of the reactor. This is a result of the neutron production  $k_{\infty}$  being a volume effect (proportional to  $r^3$  for a sphere), whereas leakage is a surface effect (proportional to  $r^2$ ). For each reactor there is a minimum *critical size* ( $k_{\text{eff}} = 1$ ) below which the surface to volume ratio is so large that neutron leakage is sufficient to prevent the fission chain reaction.

The smallest critical sizes are obtained for *homogeneous systems* of pure fissile nuclides with maximum neutron reflection. For neutrons with the fission energy spectrum, the critical mass of a metallic sphere of pure <sup>235</sup>U is 22.8 kg, that of <sup>233</sup>U is 7.5 kg, and that of <sup>239</sup>Pu is 5.6 kg, assuming a 20 cm uranium metal neutron reflector. For fission by thermal neutrons the smallest critical size of a spherical *homogeneous aqueous solution* of <sup>235</sup>UO<sub>2</sub>SO<sub>4</sub> without reflector requires 0.82 kg of <sup>235</sup>U in 6.3 l of solution. The corresponding figures for <sup>233</sup>U are 0.59 kg in 3.3 l, and of <sup>239</sup>Pu, 0.51 kg in 4.5 l.

Homogeneous solutions of fissile nuclides are produced in the reprocessing of spent fuel elements, where care must be exercised that the critical size is not exceeded in any equipment or container in order to prevent an accidental chain reaction. Several such accidents have occurred in the past in fuel production and reprocessing plants in which very high doses were received by nearby personnel even though the duration of the chain reaction usually was very short and a violent explosion never occurred.

For heterogeneous reactors it is more difficult to quote comparable simple values for critical size. These have to be calculated by numerical methods or determined empirically for each particular reactor configuration.

## **19.8. Reactor kinetics**

The mean lifetime  $\theta$  for a neutron in a reactor is the time it takes on the average for the neutrons to complete one loop in the neutron cycle. In thermal reactors  $\theta$  is  $10^{-3} - 10^{-4}$  s due to the comparatively low speed of thermalized neutrons and average distance in the moderator covered by random walk when travelling from and to a fuel pin. For each loop the number of neutrons is multiplied by a factor  $k_{\text{eff}}$ . Since one neutron is used for maintaining the chain reaction, the neutrons in the reactor change with time according to

$$dN/dt = N(k_{\rm eff} - 1)/\theta + K$$
 (19.22)

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where N is the total number of neutrons in the reactor, and K is the contribution from any constant neutron source present. Solving this equation, we obtain

$$N = N_0 e^{(k_{\rm eff}^{-1})t/\theta} + K\theta (1 - e^{(k_{\rm eff}^{-1})t/\theta}) / (1 - k_{\rm eff})$$
(19.23)

where  $N_0$  is the number of neutrons at t = 0. We can distinguish three different cases:

(i)  $k_{\text{eff}} < 1$  and  $t \gg \theta$ : the reactor is subcritical,

$$N(t) = K \Theta (1 - k_{\rm eff})^{-1}$$
(19.24)

The initial number of neutrons,  $N_0$ , has disappeared and the number of neutrons is directly related to the constant neutron source. The reactor acts as a neutron amplifier, with amplification increasing with  $k_{\text{eff}}$ .

(ii)  $k_{\text{eff}} > 1$ ; the reactor is supercritical,

$$N(t) = [N_0 - K\theta / (k_{\rm eff} - 1)] e^{(k_{\rm eff} - 1)t/\theta} + K\theta / (k_{\rm eff} - 1)$$
(19.23a)

If *K* is so small that the terms containing  $K\theta$  can be neglected this reduces to

$$N(t) = N_0 e^{(k_{\rm eff} - 1)t/\theta}$$
(19.23b)

The neutron flux (and consequently also power) increases exponentially. All reactors have K > 0, the reason being neutron production through spontaneous fission in <sup>238</sup>U and other actinides or through other nuclear reactions. However, the resulting neutron source strength is usually not large enough to give a reliable indication on the control instruments of the power level of a new reactor in the initial start-up procedure. Therefore, extra neutron sources are generally introduced in reactors to facilitate starting. A common type is Sb+ Be, where <sup>124</sup>Sb (half-life 60 d) has been produced through irradiation in a reactor using the  $(n, \gamma)$  reaction in <sup>123</sup>Sb (43% abundance). The neutrons are emitted from  $(\gamma, n)$  reactions in Be. Also <sup>241</sup>Am+ Be sources are used. When the reactor has reached desired power, control rods are inserted, which decrease  $k_{\rm eff}$  to 1. Later the partially spent fuel contains sufficient amounts of nuclides with spontaneous fission, e.g. <sup>252</sup>Cf, to make a separate neutron source superfluous.

(iii)  $k_{\text{eff}} = 1$ ; the reaction is just critical,

$$N(t) = N_0 + Kt (19.25)$$

Although this equation indicates that the number of neutrons would increase slightly with time, the *Kt* term is usually negligible.

In reactor technology it is common to speak of the *reactivity*  $\rho$  and *excess reactivity* ( $\Delta k$ ), which are defined by the expression

$$\Delta k = k_{\rm eff} - 1 = \rho k_{\rm eff} \tag{19.26}$$

Since  $k_{eff}$  is close to 1 in a properly operating reactor, we have  $\rho \approx \Delta k$ . The reactor time *constant* (or *period*) is defined as

Isotope	Half-life	decay by β-n	Cu	umulative fi	ssion yield (	%)
1	(s)	(%)	<sup>233</sup> U	<sup>235</sup> U	<sup>239</sup> Pu	<sup>241</sup> Pu
<sup>85</sup> As	2.03	23.0	0.127	0.200	0.061	0.084
<sup>87</sup> Br	55.7	2.6	2.67	1.82	0.608	0.633
<sup>88</sup> Br	16.7	6.6	1.90	1.68	0.645	0.617
<sup>89</sup> Br	4.37	14.2	0.677	1.64	0.346	0.456
<sup>90</sup> Br	1.92	24.6	0.275	0.620	0.163	0.249
<sup>93</sup> Rb	5.85	1.4	2.28	2.94	1.71	1.80
<sup>94</sup> Rb	2.73	10.4	0.849	1.54	0.754	1.22
<sup>98</sup> Y	2.0	3.4	1.64	2.99	2.59	3.58
<sup>135</sup> Sb	1.71	17.5	0.031	0.165	0.006	0.490
<sup>136</sup> Te	17.5	0.7	0.005	2.57	0.005	1.98
<sup>137</sup> I	24.5	6.4	1.35	2.60	2.46	4.56
<sup>138</sup> I	6.4	5.5	0.828	1.06	0.436	2.60
<sup>139</sup> I	2.29	9.5	0.171	0.119	0.376	1.16

TABLE 19.5. Some important fission products partly decaying by  $\beta$ -delayed n-emission

$$t_{\rm ner} = \theta/\rho \approx \theta/\Delta k = \theta/(k_{\rm eff} - 1)$$
(19.27)

As the neutron flux increases in a supercritical reactor, the second term in (19.23a) becomes negligible compared to  $N_0$  and the equation can be simplified to

$$N = N_0 e^{(t/t_{\rm per})}$$
(19.28)

The shorter  $t_{per}$  is the faster is the increase in the neutron flux (and reactor power). With  $\theta = 10^{-3}$  and  $k_{eff} = 1.1$ ,  $\Delta k$  is 0.1 and  $t_{per} \cdot 10^{-2}$ . For a longer  $t_{per}$  of 1 s, the number of neutrons would increase with a factor of  $e^{10} = 10^4$  every 10 s, which still is much too rapid for safe and simple control of a reactor. As described below, nuclear reactors are designed to avoid this problem.

In the equations above,  $\theta$  is the mean lifetime for the prompt neutrons. As discussed in §14.7.1, some fission products decay by  $\beta^-$  leading to an excited state which emits a neutron,  $\beta^-$ -delayed neutron emission. For example <sup>87</sup>Kr decays partly by  $\beta^-$ -delayed neutron emission; it is a daughter of <sup>87</sup>Br with a half-life as long as 56 s. A large number of such neutron emitting fission products have been discovered, all with shorter half-lives, see Table 19.5. In reactors where the moderator contains D or Be atoms,  $\gamma$ ,n reactions with energetic  $\gamma$ -rays from shortlived fission products and from activation products is also a source of delayed neutrons. The delayed neutrons have lower kinetic energies (~ 0.5 MeV) than the prompt ones and amount to < 1% of the total number of fission neutrons emitted: the fraction of delayed neutrons,  $\beta$ , is 0.27% for <sup>233</sup>U, 0.65% for <sup>235</sup>U, 0.21% for <sup>239</sup>Pu, and 0.52% for <sup>241</sup>Pu. When the delayed neutrons are taken into account, the *effective neutron generation time* is, approximately,

$$\theta_{\rm eff} = \theta + \Sigma \left(\beta_{\rm i} / \lambda_{\rm i}\right) \tag{19.29}$$

where  $\beta_i$  is the fraction of fission neutrons which are delayed with a decay constant  $\lambda_i$ . The summation gives the mean lifetime of the delayed neutrons; for  $^{233}U$  0.049 s,  $^{235}U$  0.084 s,  $^{239}Pu$  0.033 s, and  $^{241}Pu \sim 0.06$  s. Thus for  $^{235}U$   $\theta_{eff} \approx 10^{-3} + 0.084$ . Because the generation time determines the period of the reactor the delayed neutrons have lengthened the reactor period by almost a factor of 100, thereby making reactor control much more manageable.

 $t_{\text{per}}$  depends on  $\Delta k$  in such a way that if  $\Delta k > \beta$  the delayed neutrons are not able to make their decisive influence on the reactor period, and the rate of neutron production is dependent only on the prompt neutrons. Such a reactor is said to be *prompt critical*. Nuclear explosives are designed to be prompt critical with a  $\Delta k \ge 1$  giving a  $t_{\text{per}} \le 10^{-8}$  s, while reactors for power production (thermal as well as fast reactors) should for safety reasons always be operated in the *delayed critical* region. If in a thermal uranium reactor  $\Delta k$  is made  $\ge \beta$  ( $k_{\text{eff}} \ge 1.0065$ ),  $t_{\text{per}}$ becomes 13 s for  $\theta_{\text{eff}} = 0.084$ , and (19.28) shows that the neutron amount (and power) doubles in about 10 s. Usually  $k_{\text{eff}}$  is made smaller than 1.0065 and the power doubling time is correspondingly larger.

Some of the isobar chains from fission contain isotopes with very high n,  $\gamma$  cross sections. One of the most important is A = 135 and it's effect on reactor kinetics is used as an example. The genetic relations in the A 135 chain is:

fission yield% 3.5 2.5 0.6  

$$\stackrel{\downarrow}{^{135}\text{Te}} \stackrel{18s}{\rightarrow} \stackrel{\downarrow}{^{135}\text{I}} \stackrel{6.61}{\rightarrow} \stackrel{h}{^{135}\text{Xe}} \stackrel{9.10}{\rightarrow} \stackrel{h}{^{135}\text{Cs}} \stackrel{2 \text{ My}}{\rightarrow} \stackrel{135}{^{135}\text{Ba}(\text{stable})}$$

$$n, \gamma \downarrow 2.65 \times 10^6 \text{ b}$$

$$\stackrel{136}{^{136}\text{Xe}(\text{stable})}$$

$$n, \gamma \downarrow 0.16 \text{ b}$$

During operation at a constant neutron flux the large  $\sigma_{n,\gamma}$  continuously converts <sup>135</sup>Xe to <sup>136</sup>Xe, thereby keeping the amount small and limiting the poisoning effect. However, after a strong reduction in neutron flux <sup>135</sup>Xe formation is more rapid by decay of its precursor, <sup>135</sup>I, than its destruction, resulting in a transient radioactive equilibrium. As a result the reactivity drops for some hours because of an increasing amount of <sup>135</sup>Xe. If the reactor has been shut down for a while, it may be impossible to start it until the amount of <sup>135</sup>Xe has been reduced again by decay (*xenon poisoning*). However, if the reactor can be started it burns <sup>135</sup>Xe and  $\Delta k$  increases rapidly until a new equilibrium concentration is reached. This is called a *xenon transient* and can temporarily make the reactor difficult to control; if mishandled it may even cause prompt criticality. Once xenon poisoning has become very important after a reactor stop it is practice to delay restart until <sup>135</sup>Xe has decreased to a safe level.

#### **19.9. Fuel utilization**

Figure 19.7 shows the consumption of fissile <sup>235</sup>U while new fissile <sup>239</sup>Pu and <sup>241</sup>Pu (as well as some fission products and other actinides) are produced through radiative capture in fertile <sup>238</sup>U and <sup>240</sup>Pu. The Figure relates to a particular reactor type, and different reactors give somewhat different curves. Figure 19.5 shows the different capture and decay



FIG. 19.7. Build-up of reaction products in fuel originally enriched in  $^{235}U$  to (a) 1% used in a graphite reactor (~ 230  $MW_{th}$ ), (b) and (c) 3.3% used in a LWR (~ 1000  $MW_{e}$ ).



FIG. 19.8. Fission product balance in a light water power reactor. (From F. Hoop.)

reactions. In §19.2 it is concluded that in a reactor operating at a power of 3 GW<sub>th</sub> 3.16 kg <sup>235</sup>U d<sup>-1</sup> would be fissioned. Because of the <sup>235</sup>U(n, $\gamma$ )<sup>236</sup>U reaction, some additional uranium is consumed (19.3). To account for this, we use ( $\sigma_f + \sigma_\gamma$ )/ $\sigma_f = 1.17$ ; thus the <sup>235</sup>U consumption is 3.16 × 1.17 = 3.70 kg d<sup>-1</sup>. This value holds for a fresh reactor core; however, after a core has operated for some time, fission in <sup>239</sup>Pu formed from neutron capture in <sup>238</sup>U begins to contribute to the energy production, and, later, fission from <sup>241</sup>Pu also contributes. Therefore the <sup>235</sup>U consumption rate successively decreases for the same power production in an ageing core. During the whole lifetime of a 2 – 3% enriched uranium core about 40% of the total energy production comes from fissioning of plutonium isotopes (Fig. 19.8). On the average a 1000 MW<sub>e</sub> (33% efficiency) LWR power station consumes daily 2.2 kg <sup>235</sup>U and 2.0 kg <sup>238</sup>U. About 3.1 kg fission products are produced each day from the fissioning of uranium and plutonium. To calculate these yields, effective cross sections must be used like those given in Figure 19.5. The more <sup>239</sup>Pu and <sup>241</sup>Pu that is produced for each atom of <sup>235</sup>U consumed, the more efficient can be the utilization of fuel. This is expressed by the conversion factor *C*, which is given by

$$C = \eta - 1 - S \tag{19.30}$$

where *S* is the neutron losses by processes other than fission and capture reactions to produce fissile atoms in the fuel. If we assume C = 0.8, which is a valid figure for a heavy water moderated reactor, fission of 100 atoms of <sup>235</sup>U leads to the formation of 80 new atoms of <sup>239</sup>Pu and <sup>241</sup>Pu.

In the second generation one produces  $C^2$  new <sup>239</sup>Pu and <sup>241</sup>Pu atoms. The fraction of fissile atoms in the fuel which is transformed is  $x_i$  (1 + C +  $C^2$  + ...) =  $x_i/(1 - C)$ , where  $x_i$  is the original atomic fraction of fissile atoms (0.0072 for natural uranium). The ratio  $x_i/(1 - C)$  expresses the maximum utilization of the fuel. If a reactor could be operated so that all the fissionable material, both original and produced in operation, were consumed, it would be possible, assuming C = 0.8, to obtain five times greater power



FIG. 19.9. Conversion ratios and fuel utilization efficiencies. (From Thermal Breeder Consultants Group, Salzburg, 1977.)

production than would be provided by the original concentration of <sup>235</sup>U only. Because of the high neutron capture cross section of the fission products, unfortunately this is not possible without reprocessing between cycles. Reactors with values of *C* close to unity are called *converter reactors* while reactors with *C* considerably less than 1 are referred to as *burners*. Light water reactors usually have  $C \le 0.7$  and are classified as burners, however, designs with a reduced moderator/fuel ratio acting as converters or thermal breeders have been suggested. Figure 19.9 shows the possible fuel utilization in various reactor types.

The lowering of f (eqn. (19.13)) due to the buildup of fission products and decrease in amount of fissile atoms are the main reasons for fuel replacement. It is obvious that if  $\Sigma_a$  (fuel) is very large, as is the case for highly enriched fuel, higher amounts of fission products can be tolerated; i.e. more energy can be produced from the fuel before f becomes too small.

The fuel utilization is referred to as *burnup*. The burnup may be expressed as the percentage of fuel used before it must be replaced. For example, 1% burnup means that for each ton of fuel 10 kg of the fissile plus fertile atoms have been consumed (in fission *and* capture). However, usually the fuel burnup is given in amount of energy obtained per ton of initially present fuel atoms (in case of mixed U - Pu fuels per ton of initial heavy metal,

IHM). The production of 33 000 MWd of thermal energy (MW<sub>th</sub>d) from 1000 kg enriched uranium in a LWR consumes 25 kg  $^{235}$ U and 23.8 kg  $^{238}$ U (Fig. 19.8), or 4.88% of the original uranium amount. Then 1% burnup corresponds to 33 000/4.88 = 6762 MW<sub>th</sub>d/t U. However, if credit is made for the 6.3 kg fissile plutonium formed (which "replaces" the  $^{235}$ U), the total consumption is reduced to 48.8 – 6.3 = 42.5 kg (or 4.25%); a 1% burnup is then equal to 7765 MW<sub>th</sub>d/t U.

Considering the amount of total consumed uranium (~ 7 t natural uranium for each ton enriched to 3.3% in  $^{235}$ U), the utilization of natural uranium is much less; if plutonium is recycled, utilization increases to the overall figures given in Figure 19.9.

The maximum conversion factor at the start of a uranium fueled reactor is given by

$$C = \eta \epsilon (1 - p) e^{-B^2 \tau} + \Sigma_a (^{238} U) / \Sigma_a (^{235} U)$$
(19.31)

where the two terms give the number of neutrons absorbed by <sup>238</sup>U in the resonance absorption region and in the thermal region, respectively, per neutron absorbed in <sup>235</sup>U; the  $\eta$  value refers to <sup>235</sup>U. The term  $e^{-B^2\tau}$  is the fraction of fast neutrons which does not escape from the reactor (19.15). Using this approximate expression with the reactor example in Table 19.4, we obtain C = 0.50. For a reactor core at equilibrium corrections must be introduced for fissions in the plutonium formed; this gives a higher conversion ratio, usually ~ 0.7 in LWR.

If C > 1 the possibility exists of producing more fissile material than is consumed, which is referred to as *breeding*. The breeding ability of a reactor is given by the *breeding gain*,  $G = \eta - 2 - S$ . Table 19.2 shows that <sup>233</sup>U, <sup>235</sup>U, <sup>239</sup>Pu, and <sup>241</sup>Pu all have  $\eta > 2$ ; in practice  $\eta$  is even larger when using effective cross sections (Table 19.2 and Fig. 19.5). In a reactor with these nuclides and fertile material such as <sup>232</sup>Th and <sup>238</sup>U present it is possible to produce more fissile material than is consumed provided that neutron losses are small; such reactors are called *breeder reactors* or simply *breeders*. While the utilization of the fuel can be increased to perhaps a maximum of 10% with converters, breeding makes it possible to burn ~ 70% of the fuel material (natural uranium and natural thorium, Fig. 19.9), provided reprocessing and recycling is utilized.

From Table 19.2 it can be seen that the highest  $\eta$  value is exhibited by <sup>233</sup>U for thermal neutron energies and by <sup>239</sup>Pu for fast neutron energies. This suggests two different types of breeder reactor, the *thermal breeder* based on the reaction <sup>232</sup>Th  $\rightarrow$  <sup>233</sup>U and the *fast breeder* based on the reaction <sup>238</sup>U  $\rightarrow$  <sup>239</sup>Pu. Due to the smaller effective cross ections, these breeders must be charged with a fuel having high concentrations (15 – 30%) of <sup>233</sup>U or Pu. These materials can be produced in conventional non-breeder reactors. Because a single inelastic scattering in <sup>235</sup>U may reduce neutron energy considerably, a fast breeder should only contain little of this isotope and hence depleted uranium is preferred over natural uranium as fertile material.

For a large fast breeder reactor like the French Creys-Malville (in operation from 1986 to 1997), the fuel utilization is higher than in the LWRs; a 1% burnup corresponds to about 8500  $MW_{th}d t^{-1}$  IHM. Burnup figures as high as 15% (130 000  $MW_{th}d t^{-1}$  IHM) have been achieved in fast breeders. Breeder reactors are described in Chapter 20.



FIG. 19.10. Schematic view indicating the formation of the Oklo ore deposit and some data on its current local composition.

## 19.10. The Oklo phenomenon

At the Oklo mine in Gabon uranium has been mined for many years and delivered to the Pierrelatte isotope enrichment plant in France. The ore body, which is estimated to contain 400 000 t of uranium, is quite inhomogeneous, and pockets very rich in uranium are found embedded in sandstone and granite, see Figure 19.10. These pockets are often shaped like a lens, 10 m long and about 1 m in diameter, and contain on the average 10 - 20% pitchblende, although uranium concentrations up to 85% (pure pitchblende) are found in some spots. In 1972 it was discovered that the isotopic composition of some of the uranium received in France deviated from uranium from other sources. The <sup>235</sup>U content was



FIG. 19.11. Comparison of the isotopic composition of neodymium from normal Nd ore, from  $^{235}$ U fission in a BWR and from Oklo.

significantly lower than the natural 0.72%. Careful analysis showed that some deliveries contained  $< 0.5\%^{235}$ U, a serious disadvantage in materials to be used for <sup>235</sup>U enrichment. Analysis at Oklo showed samples even lower in <sup>235</sup>U content, as well as other elements whose isotopic composition considerably deviated from the natural one. For example, natural neodymium contains 27% <sup>142</sup>Nd, while Oklo neodymium contained < 2%, see Fig. 19.10 and 19.11. On the other hand, natural neodymium contains 12% <sup>143</sup>Nd, while in Oklo, samples containing up to 24% <sup>143</sup>Nd were found. Fission product neodymium contains about 29% <sup>143</sup>Nd, while <sup>142</sup>Nd is not produced by fission. This should be compared with the lack of <sup>142</sup>Nd and the excess of <sup>143</sup>Nd in the Oklo samples, a condition which was found to be directly related to a high total concentration of uranium but a deficiency of <sup>235</sup>U. The conclusion was obvious: in some ancient time the missing <sup>235</sup>U had fissioned, producing <sup>143</sup>Nd among other fission products. This conclusion was supported by similar investigations on the isotopic composition of other elements produced by fission.

Because <sup>235</sup>U has a shorter half-life than <sup>238</sup>U, all uranium ores were richer in <sup>235</sup>U in the past. From <sup>87</sup>Rb-<sup>87</sup>Sr analysis the age of the Oklo deposit is known to be  $1.74 \times 10^9$  y; at that time the <sup>235</sup>U content of natural uranium was 3%. Although the fission factor  $\eta$  rapidly increases with <sup>235</sup>U content (about 1.8 for 3% <sup>235</sup>U), conditions in the natural Oklo deposit were such ( $\epsilon \sim 1.0$ ,  $p \sim 0.4$ ,  $f \sim 1.0$ ) that  $k_{\infty} < 1$ . The deposit is sedimentary and was formed in the presence of water, which greatly increases the resonance escape probability factor *p*; for an atomic ratio H<sub>2</sub>O:U of 3:1,  $p \sim 0.8$ , and  $k_{\infty} > 1$ . Thus conditions existed in the past for a spontaneous, continuing chain reaction to occur in the Oklo deposit.

Further analysis and calculations have shown that these natural Oklo reactors (similar conditions occurred at several places) lasted for  $\leq 10^6$  y. Probably, criticality occurred periodically as the heat from fission boiled away the water so that the chain reaction ceased after a while. After water returned (as the temperature decreased) the chain reaction would

resume. The neutron flux probably never exceeded  $10^{13}$  n m<sup>-2</sup> s<sup>-1</sup>.<sup>1</sup> The fluence is estimated to exceed  $1.5 \times 10^{25}$  n m<sup>-2</sup>. This would have consumed about 6 t  $^{235}$ U, releasing a total energy of 2 – 3 GWy, at a power level probably  $\leq 10$  kW. About 1.0 - 1.5 t of  $^{239}$ Pu was formed by neutron capture in  $^{238}$ U, but the relatively short (on a geological scale) half-life of  $^{239}$ Pu allowed it to decay into  $^{235}$ U. Since a few samples enriched in  $^{235}$ U also have been found, it is believed that in some places in Oklo breeding conditions may temporarily have existed, cf. §22.12.

It has been calculated that many uranium rich ore deposits  $2 - 3 \times 10^9$  y ago must have been supercritical in the presence of moderating water. Therefore natural nuclear chain reactions may have had an important local influence on the early environment of earth.

### **19.11. Reactor concepts**

Nuclear reactors are designed for production of heat, mechanical and electric power, radioactive nuclides, weapons material, research in nuclear physics and chemistry, etc. The design depends on the purposes, e.g. in the case of electric power production the design is chosen to provide the cheapest electricity taking long term reliability in consideration. This may be modified by the availability and economy of national resources such as raw material, manpower and skill, safety reasons, etc. Also the risk for proliferation of reactor materials for weapons use may influence the choice of reactor type. Many dozens of varying reactor concepts have been formulated, so we must limit the discussion in this chapter to a summary of the main variables, and the most common research and power reactors. Fast reactors and some other designs are discussed in Chapter 20.

We have already mentioned three basic principles for reactor design: the neutron energy (thermal or fast reactors), the core configuration (homogeneous or heterogeneous aggregation of fuel and coolant), and the fuel utilization (burner, converter or breeder). In the homogeneous reactor the core can be molten metal, molten salt, an aqueous or an organic solution. In heterogeneous reactors the fuel is mostly rods filled with metal oxide. The fuel material can be almost any combination of fissile and fertile atoms in a mixture or separated as in the core (fissile) and blanket (fertile) concept (Ch. 20). The choice of moderator is great:  $H_2O$ ,  $D_2O$ , Be, graphite, or organic liquid. There is even more choice in the coolant which can be molten metal, molten salt, liquid  $H_2O$ ,  $D_2O$ , or organic liquid, as well as gaseous  $CO_2$ , helium or steam.

Some of the more important combinations are summarized in Tables 19.1 and 20.1.

## 19.12. Research and test reactors

According to the World Nuclear Industry Handbook 2000, 239 research and test reactors were in operation throughout the world at the end of 1999. While the smaller research reactors may have an operating power  $\leq 1 \text{ kW}_{\text{th}}$ , and do not need forced cooling, the larger test reactors operate at  $\leq 50 \text{ MW}_{\text{th}}$ . Many of these reactors have facilities for commercial radionuclide production.

<sup>&</sup>lt;sup>1</sup> n-fluxes and fluencies are given per m<sup>2</sup> in the SI system; however, it is still common to use per cm<sup>2</sup>.

Research reactors are used in nuclear physics, in nuclear, analytical and structural chemistry, in radiobiology, in medicine, etc. They are usually easy to operate, inherently safe, and of moderate cost. Many of them are of the pool type; the reactor core is located in the center of a stainless steel clad concrete vessel, 6–8 m deep and 2–5 m wide, containing purified water. The water provides the main radiation shielding, moderation, and cooling. The concrete walls are about 2 m thick when the reactor is located above ground; otherwise much thinner walls can be used. The main radiation protection demand comes from the reaction

$$^{16}O(n,p)^{16}N \xrightarrow[7.13]{\beta\gamma} {}^{16}O$$

since the  $\gamma$ 's emitted are very energetic (6.1 MeV). At power levels > 100 kW<sub>th</sub> forced cooling may be required. The fuel contains usually highly enriched uranium (20 to 90% <sup>235</sup>U) as rods or plates; the amount of <sup>235</sup>U is  $\leq$  3 kg.

Some reactors are designed to produce very high neutron fluxes, either for testing materials (especially for fast reactors) or for isotope production. The high flux reactor at Grenoble, France, has fuel plates of highly enriched (93%) UAl<sub>3</sub> canned in aluminum. The construction is a swimming pool tank type. Using 8.6 kg <sup>235</sup>U and 15 m<sup>3</sup> heavy water as moderator, it provides a maximum thermal neutron flux of  $1.5 \times 10^{19}$  m<sup>-2</sup> s<sup>-1</sup> at 60 MW<sub>th</sub>; 10 fold higher fluxes can be achieved during shorter periods. At Oak Ridge, the High Flux Isotope Reactor (HFIR) has a flux of  $3 \times 10^{19}$  m<sup>-2</sup> s<sup>-1</sup>, the highest continuous thermal neutron flux so far reported. Some research reactors are built to test new reactor designs which may be candidates for future power reactors. The successful designs lead to prototype power reactors; some of these are described in Chapter 20 together with full-scale power reactors. Other designs never survive the first stage, although the design may be very interesting.

## **19.13. Thermal power reactors**

We illustrate the general principles of thermal reactors by a short description of the two most important power reactor types; the pressurized water reactor (PWR) and the boiling water reactor (BWR). They are further discussed in Chapter 20.

#### 19.13.1. The pressurized water reactor

As the name implies a pressurized water reactor (PWR) is cooled by hot high pressure water,  $H_2O$ , which also acts as moderator. The main components of a pressurized light water moderated and cooled reactor (PWR) station have been described in §19.1 and Figure 19.2. The Ringhals station, whose first reactor started operation in 1975, is located near the sea and seawater is used for cooling; power stations located near large rivers or at the sea shore usually do not need cooling towers.

The important features of the PWR core are shown in Figure 19.12, which, although taken from three different reactors, represents the typical Westinghouse design. A typical core contains  $\sim 40\ 000$  fuel rods (a) in 193 assemblies, each with space for 208 fuel pins



FIG. 19.12. Typical PWR fuel pin (a), assembly (b, c), core (d), and vessel (e) designs.



FIG. 19.12. Continued from previous page.

(b) and (c). The fuel is 88 t of  $UO_2$  with an enrichment of 2.17% (inner region) to 2.67% (outer region) in the initial fuel loading. The reactor vessel (e) is made of stainless steel clad low alloy steel, 13 m high and 4.4 m in diameter, with a wall thickness of 22 cm; (c) shows the distribution of control rods entering from the top of the vessel. The 1060 control rods are made of a silver alloy containing 15 wt% In and 5 wt% Cd; both these elements have high thermal neutron capture cross sections. The main reactivity control is by boric acid, which is fed to the coolant through a special injection system. The boric acid circulates in the primary coolant loop, and acts as a neutron absorber. At the start of a fresh core its concentration is ~ 1500 ppm (~ 0.025 M), but it is successively reduced to zero at time of fuel replacement. The concentration is adjusted in a side loop, containing either an anion exchange or an evaporator system. The ion exchange system is so designed that  $H_3BO_3$  is fixed to the ion exchanger (the water is deborated) at low temperature (~ 30°C) while it is eluted at higher temperature (~ 80°C).

Some of the physical parameters are:  $k_{\infty}$  1.29 (cold) and 1.18 (operational temperature), temperature coefficient  $-1 \times 10^{-3} \% \Delta k/k$  °C, average neutron flux 2.16 × 10<sup>17</sup> (thermal), 3 × 10<sup>18</sup> (fast) n m<sup>-2</sup> s<sup>-1</sup>. Data for heat production are given in Table 19.6.

The Westinghouse-type PWR (Figs. 19.2 and 19.12) reactor is presently the dominating nuclear power reactor in the world.

#### 19.13.2. The boiling water reactor

In a boiling water reactor (BWR) the coolant is brought to boiling while passing along the fuel pins. Thus the need for a separate steam generator and secondary loop is removed. This gives a robust design with less complexity and marginally higher thermal efficiency than a PWR at the expense of some features. To minimize the wear of the turbine it must be fed with steam that is as dry as possible. For this purpose steam separators and steam dryers are needed. The water not converted to steam is recirculated back into the reactor core by external or internal recirculation pumps.

The single coolant – steam loop permits shortlived activity from neutron activation of oxygen in the coolant to reach the turbine. The main reaction with thermal neutrons is

<sup>18</sup>O (n, 
$$\gamma$$
) <sup>19</sup>O  $\frac{\beta, \gamma}{27.1 \text{ s}}$  <sup>19</sup>F

and with high energy neutrons

<sup>16</sup>O (n,p) <sup>16</sup>N 
$$\frac{\beta, \gamma}{7.13 \text{ s}}$$
 <sup>16</sup>O

Small amounts of fission products leaking from faulty fuel pins or generated from uranium (leached from damaged fuel or initially present as impurity) deposited on the surfaces in the core (so called *tramp uranium*) and small amounts of activation products (most troublesome is <sup>60</sup>Co) are carried by water droplets from the reactor vessel through the piping to turbine

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FIG. 19.13. Design of a boiling water reactor core system, ABB-Atom, Sweden.

and condenser. The presence of  $\gamma$ -emitters in the steam makes it necessary to shield pipes and turbines and limit access to the turbine hall during operation.

The fact that the boiling water never leaves the reactor system makes it necessary to have a special reactor water cleaning system attached. Usually this system contains mixed-bed

ion exchangers operating on partially cooled (~  $90^{\circ}$ C) reactor water. To further reduce the accumulation rate of corrosion products in the reactor the water from the turbine condenser is also cleaned by ion exchange filters before being returned as feed water to the reactor.

The reactor core and vessel of a typical boiling light water reactor station, a BWR, is shown in Figure 19.13 and supplemental data are given in Table 19.4. The fuel core and elements are rather similar to those of the PWR. The main differences are that boiling occurs in the reactor vessel, so no external steam generator is required, and that the fuel elements are enclosed in fuel boxes to improve the flow pattern. Although the cores are about the same size for PWRs and BWRs, the reactor vessel is much taller for the BWR; typical values are 22 m in height and 6 m in diameter. Since the pressure in the vessel is lower for BWRs than for PWRs, the vessel wall can be thinner (a typical value is 16 cm). BWRs usually contain more uranium, but with lower enrichment, than PWRs. While only about one-fifth of the core is replaced each year in a BWR, about one-third to one-fourth of the core is replaced annually for a PWR; in both cases about 30 t of used fuel is replaced every year.

The BWR is controlled by blades entering from the bottom of the reactor vessel. The blades are usually of cruciform cross section and fit in between the fuel boxes surrounding the fuel elements. The fuel elements often contain burnable poison (e.g.  $Gd_2O_3$ ). Because of the boiling in the tank boric acid cannot be used for reactivity control; however, it is provided as an extra emergency shutdown feature. The control blades contain boron carbide rods or powder placed in channels in the stainless steel blade-wings. In addition to negative temperature coefficients both for fuel and moderator, BWRs have a negative *void coefficient*. This means that bubble formation (*void*) along the fuel pins reduces the reactivity. As mentioned in §19.6 it is therefore common practice to control the reactor power by the main circulation pumps; increased circulation initially reduces the voids (and fuel temperature) thereby increasing power production until the previous void fraction has been almost restored at a slightly higher average fuel temperature. However, a very large negative void coefficient should also be avoided as it can cause an undesirable amplification of pressure fluctuations in the reactor vessel.

#### **19.14.** Power station efficiency

Not all of the heat produced in a nuclear reactor can be used for work, i.e. for turning the turbine blades connected to the rotor of an electric generator. According to the second law of thermodynamics, the maximum *thermal efficiency*  $(\eta)$  is

$$\eta = (Q_{\rm in} - Q_{\rm out})/Q_{\rm in} \tag{19.32}$$

where  $Q_{in}$  is the heat input into a machine and  $Q_{out}$  is the heat discharged from that machine. Most reactors operate according to the *Rankine cycle* in which a liquid medium is heated and vaporized at constant pressure, and work is carried out through adiabatic expansion. While the steam entering the turbine should be as dry as possible to reduce turbine blade wear, the steam on the backside is wet due to some condensation. This effect is normally reduced by dividing the turbine into a high pressure and a low pressure part with intermediate reheating by primary steam. Because steam is not an ideal gas (an accurate calculation of Q from heats of vaporization and the ideal gas law is impossible) empirically based steam tables must be used.

The highest thermodynamic efficiency is achieved in the *Carnot cycle* in which energy input (heating the working medium) and work both occur at different but constant temperatures,  $T_{in}$  and  $T_{out}$ . For a "Carnot engine"

$$\eta_{\rm max} = (T_{\rm in} - T_{\rm out})/T_{\rm in}$$
(19.33)

A typical PWR plant may deliver steam of ~ 275 °C to the turbine and have an outlet temperature from the condenser on the turbine backside of ~ 29 °C. Using (19.33)  $\eta_{max} = [(275 + 273) - (29 + 273)]/(275 + 273) = 0.45$ . A typical BWR usually delivers steam at ~ 285 °C and has a condenser temperature of ~ 25 °C, hence  $\eta_{max} = 0.47$ . Due to a less efficient energy cycle, friction, heat losses, pumps, etc., the net efficiency ( $\eta_{nel}$ ) of both reactor types is only about 0.32 - 0.35 (net electric output delivered to the grid divided by gross thermal output from reactor). In coal-, oil- and gas-fired power plants higher steam temperatures can be achieved, ~ 500 °C with  $T_{in}$  530 °C and  $T_{out}$  30 °C,  $\eta_{max} = 0.65$  leading to an  $\eta_{net} \sim 0.50$ .

If we compare  $\eta_{net}$  for the two kinds of power plants we find that the fossil fuel power plant has about 40% higher efficiency, i.e. the amount at "*reject heat*" is up to 40% higher for the typical nuclear power plant ((0.50–0.30)/0.50). This lower efficiency is a consequence of the lower steam temperature of the nuclear reactor. However, in the last 10 years considerable development has taken place with respect to coolant, cladding and fuel stability, and inertness, allowing considerably higher fuel temperatures, cf. Ch. 21.

The thermal efficiency of any reactor can be raised by using some of the heat directly for a beneficial purpose, such as for processing in the chemical industry or for district heating. If the diverted heat is at a higher temperature than normal for the condenser coolant effluent such diversion will reduce electricity output correspondingly. If this "lost" electricity had been used to drive heat pumps instead, the total energy efficiency had only increased by a small amount by the direct diversion of heat. In Russia some smaller nuclear power stations produce district heat and electricity for local consumption; EGP reactors in Table 19.1.

The performance characteristics of power stations are commonly described by the availability and load factors, and by the forced outage factor. The *availability factor* is the time the station has been available for operation divided by the length of the desired time period. Thus if it has been desired to operate the station for 6000 h, but it has only been possible to run it for 5500 h (because of repair, etc.) the availability factor is 92%. Typical availability factors are 95–100% for water power, 75–85% for conventional fossil power, and ~ 70% for nuclear power (usually the reactor has a higher availability, but with a lower value of ~ 70% for the turbine). Availability factors > 80% have been achieved for LWRs.

The *load factor* is the ratio between the electrical net energy delivered by the station and the net electrical energy it would have produced running at maximum power (i.e. design power multiplied by the time). A 1000 MW<sub>e</sub> power station which has produced 5.80 TWh during the year has had a load factor of 66% (=  $5.80 \times 10^6$  MWh per 1000 MW × 8760 h y<sup>-1</sup>). When nuclear power is used for *base load* (i.e. running at full power when operating), its load factor is only limited by how well the nuclear power stations perform. When nuclear power is not used for base load, the load factor depends not so much on the

power station performance but rather on the local energy demand, available alternative energy sources and their price. Thus in Sweden during 1999 the annual load factor of hydro power was  $\sim 50\%$ , thermal power  $\sim 22\%$ , wind power  $\sim 19\%$ , and nuclear power  $\sim 80\%$ . The low load factors for hydro power and thermal power are due to the use of these in "follow the load" mode, whereas nuclear power was used for base load and wind power operated at its momentary capacity. The rather low value for thermal power is caused by its cost of operation, which is higher than the other alternatives; wind power is subsidized to make it competitive. Typical world wide average yearly load factors for all power reactors weighted with production capacity are (1999 figures): PWR 76\%, BWR 82\%, PHWR 60\%, AGR 75\%, RBMK 42\% and GCR 52\%. These figures include, however, some reactors that did not operate during 1999 and thus had a load factor of zero.

A better picture of the power station performance in situations where the demand may be less than the installed capacity is given by the *forced outage rate* which is the ratio between the number of hours a power station has been shut down because of malfunction and the total number of hours during that period. A power station which has been shut off for one month in January for repair of the main feed water pump and three months (June, July, and August) because of no demand for electricity, during which time part or the fuel elements were changed, has an 8% forced outage rate (i.e. one month out of twelve). The forced outage rate is very similar for all power stations of similar size and age. Values for new 1000 MW<sub>e</sub> BWR or PWR stations vary from 15% up; a typical average value is 25%, normally decreasing with time.

## 19.15. Reactor safety

Greater than 95% of all the operation shutdowns in nuclear power stations are due to failures common to conventional thermal power stations. However, in this text we consider only safety with regard to the nuclear steam supply system which account for the other 5%. We can distinguish between three safety (or risk) levels: (i) small deficiencies due to imperfect technique, which are inevitable in any human enterprise (e.g. small "chronic" releases of radioactivity to the environment), (ii) accidents, which normally should not occur with good equipment, but still are "probable" (e.g. pipe breaks, fires, etc.) and for which protective measures must be included in the reactor design (design basis accident, DBA), and (iii) maximum credible accident (MCA) which is the most serious reactor accident that can reasonably be imagined from any adverse combination of equipment malfunction, operating errors, and other foreseeable causes. The distinction between DBA and MCA is successively vanishing, as more possible accidents are being included in the DBA. Radioactivity releases from past reactor accidents are discussed in §5.10.2 and Chapter 22.

The broad functions of the safety systems are common to most reactors. In the event of an abnormal condition they should shut down the reactor, ensure a sufficient supply of coolant for the fuel, and contain any fission products which might escape from the fuel elements. Such safety features can be *active* (requiring some action from a control system, involving mechanical devices, and relying on an external power source in order to operate) or *passive* (built-in physical fail-safe features whose operation is not dependent on any control system, mechanical device or external power source).



FIG. 19.14. Working principle of ABB-Atom SECURE inherently safe district heating reactor.

Active safety systems are normally used and must be designed with a high degree of *redundance* (duplication) and *diversity* (difference in principle), so that if one safety systems fails another shall function. Several redundant active safety systems are required which are logically and physically separated from one another, and from the reactor process systems, as much as possible. They are also often based on different principles in order to reduce the probability of common-mode failures. This confers considerable immunity against events such as external explosions or internal fires. Each safety system alone should be capable of protecting the reactor core and building from further damage.

Passive safety systems are less common than active ones. However, introduction of passive (inherent) safety functions into new reactor designs is much discussed within the nuclear industry. Several ideas have been developed into new design proposals. As an example, the SECURE district heating reactor, developed by ABB-Atom<sup>1</sup>, uses a hydro-statically metastable operating condition which causes shut-down of the reactor as soon as the normal coolant flow becomes too low for safe operation or if the core generates too much heat. Figure 19.14 illustrates the operating principle. During normal operation (a in Fig. 19.14) temperature and pressure differences keeps the system in a hydrostatically metastable state with the hot primary coolant separated from the borated shut-down solution. Power is regulated by changing the concentration of boron in the primary coolant. If either temperature in the core increases or flow through the core decreases, the gas bubble above the core will escape and the reactor shuts itself down as cold borated water

<sup>1</sup> ABB-Atom AB is now owned by Westinghouse and renamed Westinghouse-Atom AB.

enters the primary system (b in Fig. 19.14). Cooling is then secured by natural circulation inside the concrete vessel. The same principle is also proposed to be used in an inherently safe pressurized water reactor (PIUS, ABB-Atom), but many other proposed designs exist. In general, reactors with passive safety have to be designed for operation at a lower power level than reactors with active safety features.

The main threat to a reactor would involve the cooling capacity becoming insufficient because of a sudden power excursion of the reactor, a blockage of the cooling circuit, or a loss of coolant. In either case the core could become overheated and a core meltdown might begin.

The guarantee against a power excursion is a negative temperature coefficient. For example, if the design parameters of a reactor give a maximum available  $k_{eff} = 1.041$ , and the temperature coefficient is  $-4.2 \times 10^{-3} \% \Delta k/k$  °C, a temperature increase of 1000 °C would be required to reduce  $k_{eff}$  to 0.999, i.e. to automatically stop the fission chain reaction without any of the safety systems operating. Such a reactor would also be prompt critical, as pointed out in §19.8. This is not acceptable, so at starting conditions the maximum available  $k_{eff}$  is made < 1.0065 by introducing burnable poisons and control rods. After some time fission products with high neutron capture cross sections are formed (Xe, Sm, etc.), reducing  $k_{eff}$ . With  $\Delta k$  0.0065, a temperature increase of 150 °C would reduce  $k_{eff}$  to < 1 for the example given. This can be (and is) used for reactor self-regulation. The main contribution to the negative temperature coefficient in thermal reactors comes from the decreased moderator density (and also from increased void in a BWR) with increasing temperature.

Three types of control rod are used: (*control*) rods for regulating small power fluctuations, *shim rods* for coarse adjustments of  $k_{eff}$ , and *scram rods* for suddenly shutting down the reactor. The scram rods can move very fast and are actuated by gravity, spring release, compressed air, etc.; a typical value may be 3 s for complete shutdown. Since the reactor time constant is much larger, a neutron excursion is effectively stopped. The number of control rods is large ( $\geq 100$ ) in order to facilitate an even power density in the reactor, but the number of scram rods may be < 10. Boric acid dissolved in the primary coolant is normally used in PWRs for coarse reactivity adjustment (chemical shim control). In light water reactors (BWRs as well as PWRs) injection of boric acid into the core acts as an additional emergency shutdown feature ("chemical scram").

The power of a reactor does not go to zero when it is shut off. Figure 19.15 shows the heat power due to fission product decay (logarithmic time scale) after a scram. For a 1000 MW<sub>e</sub> (3000 MW<sub>th</sub>) power reactor the rest (or after) power (or heat) is 2% of the full power effect (i.e. ~ 60 MW<sub>th</sub>) after 1000 s. If this heat is not removed, the fuel and cladding would reach high temperatures. If nothing else would happen, the rather rapidly declining heat generation (Fig. 19.15), and the considerable heat capacity of the core, would probably limit the fuel temperature to a value below the melting point where the heat lost by thermal radiation and by convection in the remaining steam balances the decay heat. Unfortunately chemical reactions interfere. At normal cladding operation temperatures oxidation of zircaloy is very slow because of a protective oxide layer. However, at temperatures above ~ 1200°C the protective oxide layer fails permitting the rapid exothermic reaction between zirconium and steam according to

$$Zr(s) + 2H_2O(g) \rightarrow ZrO_2(s) + 2H_2(g) \Delta H = -6.67 + 2.57 \times 10^{-4} T (MJ/kg Zr)$$



FIG. 19.15. Decay heat from a 3000  $\rm MW_{th}$  LWR core, which has been operated continually for 300 days, as function of time after shutdown.

where T is the cladding temperature (°C). If supplied with sufficient steam, this run-away chemical reaction can rapidly heat up the fuel to its melting point resulting in a partial core meltdown. Hot molten fuel would then accumulate on the bottom of the reactor vessel. After some time the melt could penetrate the vessel bottom at the weakest spots; pouring molten core material into the containment.

The design basis accident is usually considered to start with a loss of coolant accident (LOCA), e.g. as a guillotine break in a main recirculation line. In LWRs this immediately leads to  $k_{eff} \ll 1$  because of reduced neutron moderation, independent of the position of the scram rods. However, the rest power would continue to heat the core, so core cooling is required in order to prevent a meltdown by keeping the cladding temperature below the critical range. Initiated by a lack of cooling, the heat from zircaloy oxidation caused a partial core melt-down in one of the Three Mile Island PWR reactors, Harrisburg, in 1979, but all molten material remained inside the pressure vessel. This accident had a large political impact in many countries and led to increased requirements with regard to reactor safety, e.g. improved emergency operating instructions and filtered containment venting systems.

Figures 19.16 show the *emergency cooling systems* in a BWR. There are two such systems for the core – the *core spray* and the *head spray* cooling systems (Fig. 19.16). In a modern BWR these two systems have a fourfold redundancy as Figure 19.16 shows. The systems are located in four separate zones 90° apart around the reactor and at different levels above ground. The cooling water would be taken from the wet well below the reactor. It is assumed that this well would not break; such an accident could only be initiated by an earthquake of unanticipated magnitude. In addition to the high pressure core spray there is a *low pressure coolant injection*. Further protection, especially against fission products leaking out of a damaged reactor vessel, is provided by the *containment spray*. Steam produced in the cooling is blown from the dry well into the wet well (the "blow-down" system). The water in the wet well is cooled by independent external cooling



FIG. 19.16. Emergency cooling and pressure relief system of ABB-Atom boiling water reactor (only one half of the cooling system is shown).

systems. The emergency cooling systems are much the same for BWRs and PWRs. In some plants mobile pumps can be connected from outside the containment building or water can be diverted from the fuel storage pool, thus providing an extra source of cooling water to the emergency systems.

A nuclear explosion in a reactor is physically impossible because of the large  $t_{per}$ . Rupture of the reactor vessel or pressure tubes due to overpressure is also highly improbable because of a number of safety valves (not shown in any of the Figures). The steam produced can be dumped directly into the condenser or into the containment in case of emergency. All piping into the core has isolation valves, which can be closed.

The reactor vessel is surrounded by several physical barriers. These barriers protect the surroundings from a core accident and protect the core from damage caused by external effects. In a PWR these barriers are the reactor tank (about 15-25 cm steel), the biological shield (1.5-3 m of concrete), and the outer containment. The whole primary circuit, including pressurizer dome, steam generator, and connecting pipes, is surrounded by concrete.

All nuclear power reactors outside the previous USSR and CMEA countries have a *reactor containment building*, the purpose of which is to contain steam and released radioactivities in case of a severe accident, and to protect the reactor from external damage. The containment is designed (and tested) to withstand the internal pressure from a release of the water in the entire primary cooling circuit (and in the case of PWRs of the additional loss of one of the steam generators), corresponding to excess pressures of 0.4 MPa. The containment is provided with a spray, which cools and condenses the steam released and

washes out radioactive contaminations. The steam produced in a BWR can also be dumped directly into the condenser in case of emergency. All piping into the core has insulation valves, which can be closed if desired. Modern containments are multilayered pre-stressed concrete and steel and can withstand the impact of a jumbo jet aircraft.

A special hazard, which is taken into account in the emergency systems, is that the cooling would be actuated too late, i.e. after a core meltdown has begun. In practice the cooling system must start  $\leq 1$  min after a LOCA has occurred. If the core is very hot, a violent steam and hydrogen formation could occur when the cooling water contacts the core possibly causing further heat generation by zircaloy oxidation. In this case the blow-down system, the containment, and the spray systems are still expected to contain the early energy excursion.

Under accident conditions, hydrogen might be produced from steam (or water) and zircaloy; the risk for a fire or an explosion then depends on available oxygen and mixing.

As PWR containments usually contain air, a violent hydrogen explosion could occur which would represent some hazard to a PWR with a large dry containment and even more so in case of smaller dry containments where ice is stored to be used for condensing escaping steam (ice condensers). Such containments therefore contain igniters in order to cause a controlled hydrogen burning.

In case of inerted PS-type containments, a large hydrogen production could still interfere with the blow-down process causing pressures exceeding the design limit. To prevent breaking the containment in case of overpressure all types of containments in many countries now connect by rupture disks or valves to *filtered venting systems* designed to capture most of the vented radioactive material.

In case of any event that clearly deviates form normal behavior of a reactor (ranging from observations with safety implications to catastrophes), the event should be reported to IAEA and the appropriate organizations in other countries. For reports to the press and to the general public of safety related events in nuclear power plants IAEA and OECD/NEA recommend the INES scale, see Table 19.6.

#### 19.16. Radioactive reactor waste

In the "ideal" nuclear reactor all fission products and actinides produced are contained in the fuel elements. In all practical reactors there are four processes through which radioactivity leaves the reactor vessel; in all cases the carrier of activity is the coolant:

(i) induced radioactivity in the cooling medium (and moderator if separate);

- (ii) corrosion products containing induced activities from construction materials;
- (iii) leaked fission products and actinides from faulty fuel elements;
- (iv) fission products of actinides deposited on surfaces in the core ("tramp uranium").

In all these respects pressurized and boiling water reactors present some problems of concern.

Although stringent efforts are made to contain all radioactivity formed by the nuclear reactions, it is unavoidable that small releases occur to the environment. Such releases, gaseous as well as liquid, are carefully monitored by measuring stations at the plant and at

Туре	Definition		Value	Examples	
None	No safety significance	0	0		
Incident	Anomaly		1		
	Incident		2		
	Serious incident		3	Vandellos	1989
Accident	Accident mainly in installation		4	Saint Laurent	1980
	Accident with off-site risks		5	TMI	1979
	Serious accident		6		
	Major accident		7	Chernobyl 1986	1986

TABLE 19.6. The international nuclear event scale (INES) used for the reporting of nuclear events

some locations around the plant. The authorities (nuclear regulatory commission, nuclear power inspectorate, radiation health board, etc.) set limits for such releases. Table 19.7 gives release values for a number of different reactor types and times, as reported by the United Nations. It should be noted that these are actual releases at that time. Usually the measured releases are a small fraction of the allowed limits and tend to decrease with time as more stringent limits are imposed by the regulatory bodies, improved equipment installed or better procedures adopted.

It is important to realize that the legally *permitted release* in fact sets the normal level of releases. Very much lower releases than permitted usually means increased operating costs. Hence, economic and safety considerations sets the practical release level to some chosen fraction of the legal limit, e.g. 1% or 0.1%. A 1% practical limit would give an ample margin to the legal limit, thereby reducing the risk that accidentally increased releases exceed the legal limit (which might have dire economic consequences for the plant operator).

Legal release limits are usually set so that a critical group of the general public should at most get an insignificant additional dose, typically 0.1 mSv/y. It is also customary to put a legal limit on the collective dose (usually per GW installed generating capacity) arising from operation of a nuclear power plant.

Release (kBq/k	Wh) PWR	BWR	GCR	HWR	RBMK	FBR
Airborne						
Noble gases	2.3	$3.0 \times 10^{1}$	$1.6 \times 10^{2}$	$1.3 \times 10^{2}$	$1.2 \times 10^{2}$	$3.4 \times 10^{1}$
<sup>3</sup> H	$2.7 imes10^{-1}$	$1.0 \times 10^{-1}$	$4.9 \times 10^{-1}$	$5.6  imes 10^1$	3.0	5.6
<sup>14</sup> C	$1.9  imes 10^{-2}$	$5.5  imes 10^{-2}$	$1.1 \times 10^{-1}$	$3.7  imes 10^{-1}$	$1.5  imes 10^{-1}$	$1.4 \times 10^{-1}$
<sup>131</sup> I	$2.9  imes 10^{-5}$	$6.3  imes 10^{-5}$	$1.0  imes 10^{-4}$	$2.9  imes 10^{-5}$	$8.0  imes 10^{-4}$	$2.9 \times 10^{-1}$
Particulates	$1.7  imes 10^{-5}$	$3.0  imes 10^{-2}$	$2.9  imes 10^{-5}$	$5.7 \times 10^{-6}$	$1.3  imes 10^{-3}$	$7.4 \times 10^{-1}$
Liquid						
<sup>3</sup> H	2.3	$1.0 \times 10^{-1}$	$2.9 \times 10^{1}$	$4.7 \times 10^{1}$	1.3	$2.0 \times 10^{-1}$
Other	$1.5  imes 10^{-3}$	$3.1 \times 10^{-3}$	$6.9 \times 10^{-2}$	$9.9 \times 10^{-3}$	$6.3  imes 10^{-4}$	$4.1 \times 10^{-1}$

TABLE 19.7. Average atmospheric and aquatic releases of radionuclides from various reactor types 1990-1997 (Data from UNSCEAR 2000)

## 19.17. Nuclear explosives

Assume that 50 kg of metallic  $^{235}$ U is brought together to provide a critical configuration. Using a generation time of  $3 \times 10^{-9}$  s for neutrons, it can be calculated with (19.27) and (19.28) that it would take  $0.2 \times 10^{-6}$  s to increase the number of neutrons from one to that required for fissioning all  $^{235}$ U atoms. However, long before this time, the energy absorbed in the material would have blown it apart, so 100% efficiency is impossible in a nuclear explosion. If 2% of the 50 kg of uranium had been fissioned, the amount of energy released corresponds to about 20 000 t of TNT (1 t of TNT corresponds to  $4.19 \times 10^9$  J). The energy production is said to be 20 kt. Only  $^{233}$ U,  $^{235}$ U, and  $^{239}$ Pu (of the more easily available longlived nuclides) have sufficiently high cross sections for fission by fast neutrons to have a reasonable critical mass for use as nuclear explosives.

A rough estimate of the critical radius of a homogeneous unreflected reactor may be obtained simply by estimating the neutron mean free path according to (14.6). Assuming  $^{235}$ U metal with a density of 19 g cm $^{-3}$  and a fast fission cross section of  $2\times10^{-24}$  cm $^2$ , one obtains  $\Sigma_f^{-1}=10$  cm. A sphere with this radius weighs 80 kg. For an unreflected metal sphere containing 93.5%  $^{235}$ U the correct value is 52 kg.  $^{239}$ Pu has the smallest unreflected critical size; for  $^{239}$ Pu ( $\delta$ -phase, density 15.8 g cm $^{-3}$ ) it is 15.7 kg ( $\sim$  6 kg reflected), and for  $^{233}$ U 16.2 kg ( $\sim$  6 kg reflected).

Assume two half-spheres of metallic  $^{235}$ U, each of which is less than critical size but which together exceed criticality. As the distance between the two spheres is diminished the  $k_{eff}$  increases for the total system. At a certain distance  $k_{eff}$  can be maintained close to 1 but the chain reaction still be prevented. If the distance between the two half-spheres is diminished rapidly to less than this critical distance, a single neutron can trigger a rapidly multiplying chain reaction. Since stray neutrons are always available, e.g. from spontaneous fission, a chain reaction would be started when the distance between the spheres is less than the critical distance.

Nuclear fission weapons contain "normally" either highly enriched <sup>235</sup>U (> 93%) or fairly pure <sup>239</sup>Pu ( $\ge$  95%). Some early nuclear explosives operated by having two subcritical parts of a sphere blown together by the use of chemical explosives (*gun type*). To increase the explosive yield the fissile material is often surrounded by a heavy material (*tamper*) which has the double function of reflecting the neutrons and of acting as a heavy mass (inertia) to increase the time in which the supercritical configuration is held together. If natural uranium is used as the tamper, secondary fission in this shell can contribute to the overall energy release. However, only a fraction of the fission neutrons have energies large enough to fission <sup>238</sup>U efficiently.

A critical mass of material can also be arranged as a subcritical spherical shell which can be compressed into the supercritical sphere. This process is called *implosion* and is probably used in most weapons. The bomb over Hiroshima ( $\sim 15$  kt) used uranium and was of the gun type, while the Nagasaki bomb ( $\sim 22$  kt) was of the more efficient implosion type using plutonium.

In order for these devices to function efficiently, the neutron multiplication must not begin until the critical size has been well exceeded. Furthermore, the number of neutrons in the initial generation must be sufficiently high in order to reach a very high n-flux in the short lifetime of the supercritical configuration. Therefore, a few microseconds before maximum criticality a large number of neutrons is injected by a special neutron source initiating a rapid neutron multiplication.

The amount of <sup>240</sup>Pu in a plutonium charge must be maintained relatively low since this nuclide decays partly through spontaneous fission with the resultant release of neutrons. If the concentration of <sup>240</sup>Pu is too high, the neutron multiplication will start too early and with too few neutrons in the first generation, and the energy release is decreased. Also <sup>241</sup>Pu, if present in larger percentage, could cause problems because of its rather rapid decay to <sup>241</sup>Am. To minimize the <sup>240</sup>Pu and <sup>241</sup>Pu content, plutonium for weapons used to be produced in special reactors by low burnup (< 2000 MW<sub>th</sub>d t<sup>-1</sup>); this is uneconomical to the nuclear power industry where high burnup of the fuel (leading to higher <sup>240</sup>Pu concentrations; Fig. 19.7) is a necessity in order to obtain the lowest electricity production costs. It has been stated that also power reactor grade plutonium, even as oxide, can be used in a weapon, though less effective and predictable. The critical mass for such plutonium containing 20% <sup>240</sup>Pu has been estimated to be 20 kg for metal and 37 kg for PuO<sub>2</sub>.

Nuclear weapons of the fission type have been developed in sizes ranging from 0.001 kt to about 500 kt yield. Because the critical amount can never be less than several kilograms of fissile material, a weapon of low yield such as 0.001 kt has very low efficiency ( $< 5 \times 10^{-6}$ ). The highest achieved efficiency of a plutonium based charge seems to be an order of magnitude higher than that achieved by a <sup>235</sup>U based charge. However, most fission devices are assumed to contain both materials. Seven countries (USA, USSR, UK, France, China, Pakistan, and India) have carried out > 1000 known nuclear weapons tests (fission and fusion devices). Since the Partial Test Ban Treaty in 1963, most explosions have been underground. At the time of writing there is an agreement to ban furter tests.

In the explosion of fission weapons a temperature of approximately  $10^8$  K is obtained. This temperature is sufficient for producing fusion reactions between deuterium and tritium (see Ch. 17). Although no official information is available on how modern fusion weapons (hydrogen bombs) are constructed, it is generally assumed that they are based on the principle of using a fission charge as the initiator of the fusion reaction. In the debris of hydrogen bombs, lithium has been discovered. It is therefore believed that in hydrogen bombs, deuterium is combined with lithium in the form of solid LiD. <sup>6</sup>Li will react with a neutron to form tritium in the reaction

$$^{6}\text{Li} + n \rightarrow T + {}^{4}\text{He} \quad Q = 4 \text{ MeV}$$

With fast neutrons the reaction

$$^{7}\text{Li} + n(\text{fast}) \rightarrow T + {}^{4}\text{He} + n$$

also occurs.

The deuterium and the tritium can react as shown by equations (17.15)-(17.19). If the temperature and pressure is high enough the D-D reaction can also contribute. Natural uranium, depleted uranium, or enriched uranium must probably be used to improve the neutron economy when the charge contains LiD. Secondary fission in uranium also leads to an increased energy production in the weapon. The high energy of the neutrons from the fusion reactions make them very effective for fission of <sup>238</sup>U. The fission of <sup>238</sup>U produces

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large amounts of radioactive fission products, resulting in a "dirty" weapon. By contrast, if a tamper of a nonfissionable heavy material is used the only fission products released are those obtained from the ignition process and the weapon is relatively "clean".

Charges of sizes up to  $\sim 60\ 000$  kt TNT have been tested. If it is assumed that half of the energy of a 50 Mt device comes from fusion and the other half from fission, and a 25% efficiency in the explosion, about 8 t of nuclear material is required.

Very high temperatures are reached in the center of a nuclear explosion (~  $10^8$  K). The exploded material forms a fireball which rapidly expands and moves upward in the atmosphere. After the first wave has passed, a low pressure area is formed under the fireball which draws material from the ground to produce the typical mushroom-like cloud associated with nuclear explosions. This cloud contains fission products as well as induced radioactivity from the construction material of the device and from the immediate surrounding ground that has been irradiated. Some of the radioactive particles are carried to great heights and eventually fall from the atmosphere as *radioactive fallout*, see Ch. 22. Much of the fallout returns to the earth close to the explosion site, but some of the material reaches the stratosphere and returns to the earth only slowly over a long period of time and over great distances from the original explosion site. The radioactivity of the fallout decays with time according to the approximate relationship

$$A = A_0 e^{-1.2t} (19.35)$$

where  $A_0$  is the radioactivity immediately after the explosion and *t* is the time in hours after the explosion.

The energy produced in the explosion of a 20 kt weapon is distributed approximately as follows: 50% pressure; 35% heat; 5% instantaneous radiation; 10% radiation from fission products. The radiation dose at 500 m from hypocenter (the vertical point on ground below the explosion center) has been estimated to be  $\sim$  70 Gy; at about 1.1 km it is  $\sim$  4 Gy. The pressure wave moves from the explosion site with the velocity of sound. This initial short pressure front is followed by a low pressure wave leading to a rapid change in wind direction. The deaths in Hiroshima and Nagasaki resulted in 20 – 30% of the cases from primary burns, 50 – 60% from mechanical injuries and secondary burns, and only  $\sim$  15% from radiation injuries. The health detriment to the Japanese population is discussed in Chapter 18.

Table 1.1 lists some historical "firsts" with regard to nuclear weapons. The extensive tests in the atmosphere up to 1963 lead to a large global spread of tritium, <sup>14</sup>C, fission products and actinides. Scientists have used this to learn more about global wind and water currents. Radiochemists have studied the migration of deposited radionuclides, as discussed in Chapter 22, radioecologists the uptake of radioactive elements by plants and animals, as described in Chapter 18, etc.

A number of ideas have been proposed for the peaceful use of nuclear explosives. It has been estimated that the explosive power of nuclear charges is sufficiently great that their use for large scale explosions would be much cheaper than the use of conventional explosives. About 136 explosions have been used for civil engineering projects in the former USSR; sizes varied from < 1 kt to ~ 50 kt. In the United States crater formation and gas stimulation experiments have been made. Suggested uses include the blasting of harbors on remote coasts to allow greater access to inland mineral deposits, crushing of ore

bodies in order to obtain valuable minerals, crushing of underground rocks which may hinder the use of natural gas reservoirs, diverting asteroids threatening the earth, etc.

#### 19.18. Exercises

**19.1.** If the energy developed by a 20 Mt fusion weapon could be used for producing electricity at a value of 2 cents kWh<sup>-1</sup> what would the "electric value" of the device be? Such a weapon may be expected to cost \$ 10 million. One ton TNT releases 1 Gcal of energy in an explosion.

**19.2.** The bomb over Hiroshima contained <sup>235</sup>U. How many grams were fissioned to correspond to 15 kt of TNT?

19.3. Compare two 500 MWe electric power stations, one burning oil and the other using 3.0% enriched uranium. Both stations operate 6000 h  $y^{-1}$  at 35% efficiency (heat to electricity). The oil (43.5 MJ/kg combustion energy) is carried by 100 000 ton d.w. (dead weight, i.e. carrying capacity) oil tankers, and the uranium fuel by train cars of 20 t capacity each. (a) How many oil tankers will be needed every year for the oil-fired station? How many train cars will be needed every year for the nuclear power station for transporting (b) the enriched UO<sub>2</sub> reactor fuel, (c) the corresponding amount of natural uranium as  $U_3O_8$  to the isotope enrichment plant, if the tail is 0.35% in <sup>235</sup>U? (See Ch. 2 and §19.9.) Reactor fuel rating 40 MW<sub>th</sub>d/kg U.

**19.4.** Calculate the number of collisions required to reduce a fast fission neutron ( $\tilde{E}_n^0 = 2$  MeV) to thermal energy ( $E_n$ 0.025 eV) in a light-water-moderated reactor, assuming that the data in Table 19.3 are valid.

19.5. Calculate the thermal fission factor for a mixture of 60%  $^{239}Pu$ , 30%  $^{240}Pu$ , and 10%  $^{241}Pu$ .

19.6. The world's first water boiler reactor (LOPO, Los Alamos, 1944) was a homogeneous solution of enriched uranium sulfate as follows: 580 g <sup>235</sup>U, 3378 g <sup>238</sup>U, 534 g S, 14068 g O, and 1573 g H. From these values, and Tables 19.2 and 19.3, calculate  $\eta$  and f; neglect S. With p = 0.957, what will  $k_{\infty}$  be?

**19.7.** A large homogeneous thermal reactor contains only  $^{235}$ U dispersed in beryllium in the atomic ratio  $1:3 \times 10^4$ . The migration area is 0.023 m<sup>2</sup>. Assuming  $p = \epsilon = 1$ , calculate the size of a cylindrical reactor with height equal to diameter.

19.8. A cubic unreflected graphite moderated natural uranium reactor contains 3% enriched uranium as UC homogeneously dispersed in the graphite matrix; the weight ratio C/U = 10. The resonance passage and thermal utilization factors are both assumed to be 0.9;  $\epsilon = 1.00$ . Make an estimate of the critical size of the cube.

**19.9.** The LOPO reactor in exercise 19.6 has a neutron age  $\tau = 31.4$  cm<sup>2</sup>, and diffusion area  $L^2$  1.87 cm<sup>2</sup>. Calculate (a) the fast neutron leakage factor, and (b) the critical radius for the homogeneous sphere, if  $k_{\infty} = 1.50$ . **19.10.** Our solar system is considered to be 4.5 billion years old. What was the <sup>235</sup>U percentage in natural uranium

when the solar system was formed?

**19.11.** The radiometric sensitivities for discovering  ${}^{59}$ Fe,  ${}^{131}$ I, and  ${}^{90}$ Sr are 75, 25, and 0.74 kBq m<sup>-3</sup> of water. In the Würgassen plant the total permitted aqueous annual release was 17 Ci β-emitters. Assume an activity ratio in the cooling water of 100:10:1 for the three nuclides above and that none of these activities exceed 1% of the permissible release. How many times must a liquid sample taken each day be concentrated to meet these requirements?

19.12. What amount of tritium (Bq) is produced in the Würgassen nuclear plant assuming that <sup>3</sup>H is only produced through capture in the deuterons of the original cooling water, the amount or which is 50% of the core volume? Data on fluxes, cross sections, and releases are given in Table 19.4. Neglect the tritium decay rate.

19.13. A BWR has operated at full power for a week. At what time after a scram would the Xe poisoning reach its maximum? Use data in Table 19.4.

**19.14.** The net efficiency of a 1000 MW<sub>o</sub> BWR is 34.1%. Estimate the amount of <sup>235</sup>U consumed during its first day of full power operation with a completely fresh load of fuel.

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